Hypothesis Testing Procedures

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Hypothesis Testing Procedure

We will assume at first that the population standard deviation σ is known.

The procedure when σ is unknown is very similiar and a separate procedure will be provided for that situation.

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- We have a random sample of size n from the population
- We know the sample mean $\overline{x}$
- We know that if the null hypothesis H_0 is true, the population mean is μ_0.
- We have chosen a value α for the level of significance
- We are interested in a Two-Tailed test: The alternative hypothesis H_1 states that $\mu \neq \mu_0$, the population mean μ *does not* equal μ_0.

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- Otherwise, reject H_0 in favor of H_1.

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$Z_{\alpha/2}$ can be obtained from the standard normal distribution table, or from a spreadsheet using the formula

$$= NORMSINV(1-\alpha/2)$$

(in the actual formula, $-\alpha/2$ would be replaced by a value or a cell reference).

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The Z score for the sample mean $\overline{x}$ is:

$$Z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$$

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then accept H_0: $\mu = \mu_0$

If the Z score lies **outside** the confidence interval, that is,

$$L \ > \ Z \quad \text{or} \quad U \ < \ Z$$

then reject H_0 in favor of $H_1 : \mu \neq \mu_0$.