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The same problems we mentioned in connection with sampling for proportions are present.

As a result, sampling is often the only viable option.

As before, it is important to determine the accuracy of the results.

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If you sample from a small population, it can make a difference.

There are ways of adjusting the results to allow for this.

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Common IQ tests are standardized to have a standard deviation of 15.

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$$SD_{\bar{x}} = \frac{SD_x}{\sqrt{n}}$$

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It is a fact that the  $t$  distribution becomes indistinguishable from the bell curve as the sample size grows.