3.6 Exercises Problem 2

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A *r.v.* X has mean $\mu \neq 0$ and variance $\sigma^2 > 0$. If the third moment of X about μ is $-\mu^3$, what is E(X^3)?

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$$\mathsf{E}(X-\mu)^3$$

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Expanding the cubic gives

$$\mathsf{E}(X-\mu)^3 \;=\; \mathsf{E}(X^3 - 3X^2\mu + 3X\mu^2 - \mu^3)$$

$$=\; \mathsf{E}(X^3) - 3\mu\mathsf{E}(X^2) + 3\mu^2\mathsf{E}(X) - \mu^3$$

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Adding μ^3 to both sides,

$$0 = \mathsf{E}(X^3) - 3\mu\sigma^2 \quad \Rightarrow \quad \mathsf{E}(X^3) = 3\mu\sigma^2$$