AVERBACH AND MEHTA 3.6 EXERCISES - #17

17) Let X and Y be *r.v.'s* with $\mathrm{Var}(X) = 4$, $\mathrm{Var}(Y) = 9$, and $\mathrm{Var}(X - Y) = 16$. What is $\mathrm{Cov}(X, Y)$?

a) -3/2 b) -1/2 c) 1/2 d) 3/2 e) 13/16

Solution: If $\mathrm{Cov}(X, Y) = \sigma_{XY}$ the variance-covariance matrix of $[X, Y]$ is

$$V = \begin{bmatrix} 4 & \sigma_{XY} \\ \sigma_{XY} & 9 \end{bmatrix}$$

and the transform vector is

$$a = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

and the variance of $(X - Y)$ is

$$\begin{aligned} a'Va &= \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 4 & \sigma_{XY} \\ \sigma_{XY} & 9 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ &= \begin{bmatrix} 4 - \sigma_{XY} & \sigma_{XY} - 9 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \\ [13 - 2\sigma_{XY}] &= 16 \\ -2\sigma_{XY} &= 3 \quad \text{so} \quad \sigma_{XY} = -\frac{3}{2} \end{aligned}$$