3.6 Exercises Problem 1

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3.6 Problem 1

Let X be a *r.v.* with pdf

$$ae^{-ax}, \quad a, x > 0$$

If $M_x(t)$ is the mgf of X, what is $M_x(-3a)$?

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By definition,

$$M_x(t) = \mathsf{E}(e^{tx})$$

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For the given density function (which is the exponential distribution),

$$M_x(t) = \mathsf{E}(e^{tx}) = \int_0^\infty e^{tx} \cdot ae^{-ax}\,dx$$

$$= \int_0^\infty ae^{(t-a)x}\,dx = -\frac{a}{t-a}e^{(t-a)x}\Big|_0^\infty$$

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Assuming $t < a$, the integral converges to

$$-\frac{a}{t-a} = \frac{a}{a-t} = M_x(t)$$

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Substituting $t = -3a$,

$$M_x(-3a) = \frac{a}{a-(-3a)} = \frac{a}{4a} = \frac{1}{4}$$