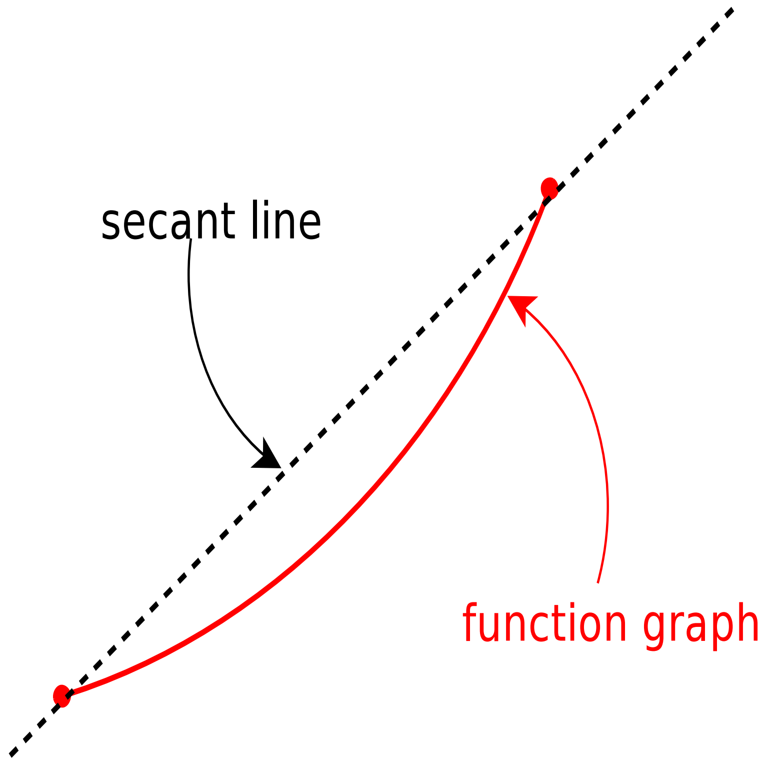
# SECANT LINES

## 1. Secant Lines

By a **secant line**, we mean a straight line connecting two different points on a curve.

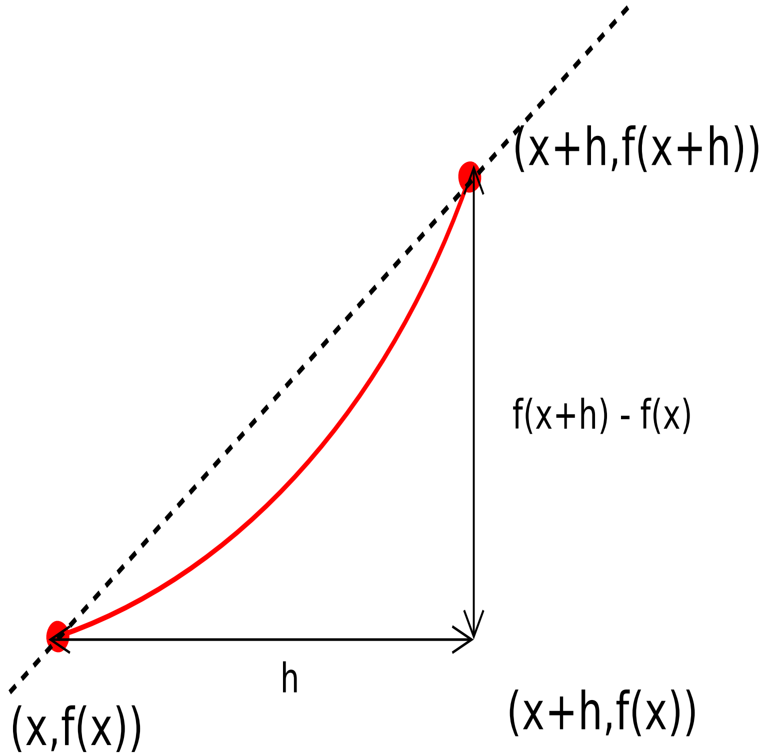



There are two points of intersection between the line and the function graph, which we will label

$$(x, f(x)) \quad \text{and} \quad (x+h, f(x+h))$$

for some positive value $h$.

Then we can label the graph as follows:




The slope of the secant line is:

$$\text{slope} \quad = \quad \frac{\text{rise}}{\text{run}} \quad = \quad \frac{f(x+h)-f(x)}{(x+h)-x} \quad = \quad \frac{f(x+h)-f(x)}{h}$$

**Example 1.1.** *Find the slope of the secant line that intersects the graph of the function*

$$y = x^2$$

*at the points* $(1,1)$ *and* $(2,4)$. *Identifying the points of intersection with those in the graph, namely* $(x, f(x))$ *and* $(x+h, f(x+h))$, *we have*

$$(x, f(x)) \ = \ (1,1) \quad \textit{and} \quad (x+h, f(x+h)) \ = \ (2,4)$$

*The slope of the secant line is then:*

$$\textit{slope} \quad = \quad \frac{f(x+h)-f(x)}{(x+h)-x} \quad = \quad \frac{4-1}{2-1} \quad = 3$$

**Example 1.2.** *Find the slope of the secant line that intersects the graph of the function*

$$y = x^2$$

*at the points* $(0,0)$ *and* $(2,4)$*. Identifying the points of intersection with those in the graph, namely* $(x, f(x))$ *and* $(x+h, f(x+h))$*, we have*

$$(x, f(x)) \ = \ (0,0) \quad \textit{and} \quad (x+h, f(x+h)) \ = \ (2,4)$$

*The slope of the secant line is then:*

$$\textit{slope} \quad = \quad \frac{f(x+h) - f(x)}{(x+h) - x} \quad = \quad \frac{4-0}{2-0} \quad = \frac{4}{2} \ = \ 2$$

Evidently, the slope of the secant line depends on the values of $x$ and $h$. It would be helpful to have a general formula in terms of $x$ and $h$.

If $f(x) = x^2$, then we can write

$$\text{slope} \quad = \quad \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$= \quad \frac{(x+h)^2 - x^2}{x+h-x} \quad = \quad \frac{(x^2 + 2xh + h^2) - x^2}{h} \quad = \quad \frac{2xh + h^2}{h}$$

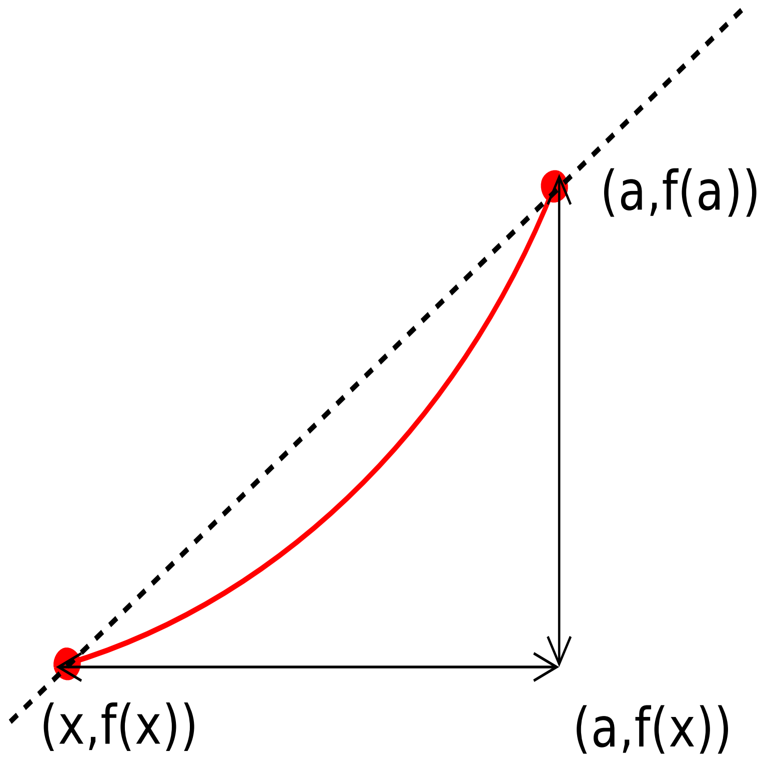
So the slope of the secant line joining $(x, f(x))$ and $(x+h, f(x+h))$ has slope

$$\frac{f(x+h) - f(x)}{(x+h) - x} \quad = \quad \frac{2xh + h^2}{h} \quad = \quad 2x + h$$

Now we can redo our first example: Find the slope of the secant line to $f(x) = x^2$ between $(1,1)$ and $(2,4)$ Identifying $x = 1$ and $h = 1$, the slope of the secant line is $2 \cdot x + h \ = \ 2 + 1 \ = \ 3$ In the second example, $x = 0$ and $h = 2$, so the slope of the secant line is $2 \cdot 0 + 2 \ = \ 2$.

1.1. **Alternative Notation.** An alternative notation for designating the secant line between $(x, f(x))$ and $(x+h, f(x+h))$ is

$$(x, f(x)) \quad \text{and} \quad (a, f(a))$$




Then the slope of the secant is

$$\text{slope} \quad = \quad \frac{f(x) - f(a)}{x - a}$$

**Example 1.3.** *Find the slope of the secant line to the graph of the function $f(x) = \sqrt{x}$ from $x = 0$ to $x = 2$.*

$$slope \quad = \quad \frac{f(2) - f(0)}{2 - 0} \quad = \quad \frac{\sqrt{2} - \sqrt{0}}{2} \quad = \quad \frac{\sqrt{2}}{2}$$