## 1. Project 6

The *Beverton-Holt* model

$$x_{n+1} = \frac{rKx_n}{K + (r-1)x_n}, \quad K > 0,\ r > 0$$

has been used to model populations of North Atlantic bottom-feeding fish species such as haddock that have high fertility rates and low rates of survival to adulthood.

**Problem 1:**

Part 1) Determine the equilibrium solution(s) for this model, if any exist.

Part 2) If there are equilibria, for each one determine the parameter values, if there are any, for which it is asymptotically stable.

**Problem 2:**

Suppose a certain proportion of the population $\alpha$ is harvested. The modified *Beverton-Holt* model is:

$$x_{n+1} = \frac{rKx_n}{K + (r-1)x_n} - \alpha x_n, \quad K > 0,\ r > 0,\ 0 \leq \alpha < 1$$

Use $K = 100$ to represent 100% of the carrying capacity, and $r = 1.3$.

Part 1) Determine the positive equilibrium solution for this model when $\alpha = 0$ and $0 < \alpha < 1$.

Part 2) Determine the parameter values for which the positive equilibrium is asymptotically stable.

Part 3) How large can $\alpha$ be before the positive equilibrium approaches zero?

Part 4) Define a sustainable catch as $\alpha$ times the equilibrium population size when the latter is positive. With $K = 100$ and $r = 1.3$, what is the maximum sustainable catch, and the value of $\alpha$ at which it occurs?