1. ASSIGNMENT 2

1.1. Problem 1.

1.1. Problem 1. Suppose $Y = (Y_1, \dots, Y_n)$ is a vector of IID $N(0,1)$ random variables.

a) Find the distribution of

$$\sum_{i=1}^{n} Y_i^2$$

b) Define

$$W_n = \frac{1}{n}\sum_{i=1}^{n} Y_i^2$$

Show that W_n converges in probability to a constant, and find the value of the constant.

1.2. Problem 2. Suppose $Y = (Y_1, \dots, Y_n)$ is a vector of IID $N(\mu,1)$ random variables.

a) Show that Y_1 is an unbiased estimator for μ

b) Find

$$P(|Y_1 - \mu| \le 1)$$

c) Based on the result of b), is Y_1 a consistent estimator for μ?

1.3. Problem 3. Suppose Y is a random variable with a binomial distribution having n trials and probability of success p. Show that

$$\frac{1}{n}Y$$

is a consistent estimator for p.

1.4. Problem 4. Let $Y = (Y_1, \dots, Y_n)$ be a random sample of size n where each Y_i has the density function

$$f(y) = \begin{cases} \theta y^{\theta-1} & \text{if} \quad 0 < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

where $\theta > 0$. Show that

$$\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \quad \text{is a consistent estimator of} \quad \frac{\theta}{\theta+1}$$