1. ASSIGNMENT 1

1.1. **Problem 1.** Suppose Y is a random variable with support on $[0, \infty)$ and density function $f_y(y)$. Define an new random variable

$$Z = g(Y) \quad \text{where} \quad X = g(Y) = kY, \quad k > 0$$

a) Find the density function of Z, $f_z(z)$.

b) Use the density functions of Y and Z to show that for $b > 0$,

$$P(0 \leq Y \leq b) \;=\; P(0 \leq Z \leq kb)$$

1.2. **Problem 2.** Suppose $Y = (Y_1, \ldots, Y_n)$ is a vector of IID exponential random variables with parameter θ, and let $S = \sum_{i=1}^{n} Y_i$.

a) Find the density function of S (hint: use moment-generating functions)

b) Find the expected value and MSE of

$$\hat{\theta} = \frac{S}{n}$$

1.3. **Problem 3.** Suppose $Y = (Y_1, \ldots, Y_n)$ is a vector of IID exponential random variables with parameter θ, and let

$$Y_{(1)} = \min(Y_1, \ldots, Y_n)$$

a) Find the density function of $Y_{(1)}$

b) Find expected value and MSE of

$$\hat{\theta} \;=\; n \cdot Y_(1)$$

1.4. **Problem 4.** Suppose $\hat{\theta}_1$ and $\hat{\theta}_2$ are estimators of θ with

$$E(\hat{\theta}_1) = E(\hat{\theta}_2) = \theta, \quad V(\hat{\theta}_1) = \sigma_1^2, \quad \text{and } V(\hat{\theta}_2) = \sigma_2^2$$

Find the value of the constant a that minimizes

$$V(\hat{\theta}_3) \quad \text{where} \quad \hat{\theta}_3 \;=\; a \cdot \hat{\theta}_1 + (1 - a) \cdot \hat{\theta}_2$$