1. The Weibull Distribution

The Weibull distribution is sometimes characterized as an ”extreme value” distribution because it can have a long tail depending on the parameters.

This makes it useful for modelling data that contains large values that would be very unlikely with, say an exponential distribution.

The density function is:

$$f(x) = \frac{a}{b}\left(\frac{x}{b}\right)^{a-1} \exp\left(-\frac{x}{b}\right)^{a} \quad x > 0$$

The expected value and variance are:

$$E(X) = b \cdot \Gamma\left(1 + \frac{1}{a}\right)$$

and

$$V(X) = b^2 \cdot \left(\Gamma\left(1 + \frac{2}{a}\right) - \Gamma\left(1 + \frac{1}{a}\right)\right)^2$$

2. The Logistic Distribution

The Logistic distribution is another ”extreme value” distribution because it can have a long tail depending on the parameters.

This makes it useful for modelling data that contains large values that would be very unlikely with another distribution.

The density function is:

$$f(x) = \frac{1}{b}\left(\frac{\exp\left(\frac{x-a}{b}\right)}{\left[1 + \exp\left(\frac{x-a}{b}\right)\right]^2}\right) \quad x > 0$$

The expected value and variance are:

$$E(X) = a$$

and

$$V(X) = \frac{2b^2\pi^2}{6}$$

3. The Lognormal Distribution

The Lognormal distribution describes a random variable whose natural log has a normal distribution.

The lognormal is also useful for modelling data with large deviations.

The density function is:

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma x} \exp\left(-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right) \qquad x > 0$$

The expected value and variance are:

$$E(X) = \exp\left(\mu + \frac{1}{2}\sigma^2\right)$$

and

$$V(X) = \exp\left(2\mu + \sigma^2\right) \cdot \left(\exp(\sigma^2) - 1\right)$$