

1. ASSIGNMENT 8

1.1. **Problem 1.** Suppose X and Y are random variables with joint probability density function (joint pdf)

$$f(x, y) = \frac{x^2 + y^2}{26} \quad 0 \leq x \leq 2, \quad 0 \leq y \leq 3$$

- a) Show that f is a valid joint density function (i.e., show that $f(x, y) \geq 0$ and the integral of f over its support is one)
- b) Find the joint cumulative distribution function (CDF) of X and Y
- c) For any function $u(x, y)$,

$$E(u) = \int_0^2 \int_0^3 u(x, y) f(x, y) dy dx$$

Find the expected values $E(X)$ and $E(Y)$ and the variance-covariance matrix of X and Y , defined by:

$$V_{XY} = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{bmatrix}$$

1.2. **Problem 2.** Using the joint density function from problem 1),

- a) Find the marginal PDFs of X and Y and show that they are valid.
- b) Find the marginal CDFs of X and Y
- c) Find the expected values $E(X)$ and $E(Y)$ and the variances of X and Y using the marginal PDFs. How does your answer compare with 1c?

1.3. **Problem 3.** Using the joint density function from problem 1),

- a) Find the conditional PDFs of X given Y and Y given X . Show that they are valid.
- b) Find the conditional expected value and variance of X given Y and Y given X .
- c) Find conditional expected value of X given that $Y = 1$, and the conditional expected value of Y given that $X = 1$.

1.4. **Problem 4.** Suppose X and Y are random variables having a bivariate Bernoulli distribution with the following probabilities:

X:	Y:0	1
0	p_{00}	p_{01}
1	p_{10}	p_{11}

- Find the marginal density functions of X and Y .
- Find the conditional density functions of X given Y and Y given X .
- Find the expected values of X and Y and their variance-covariance matrix V_{XY} .