1. Written Assignment 1

The coin toss experiment has two outcomes, H and T. In the following problems, the experiment consists of repeating the coin toss three times.

The assignment is to construct a probability space $S = (\Omega, \mathcal{F}, \rho)$ that represents the experiment, and to define a random variable X on it.

1.1. **Problem 1.** What is the sample space Ω for this experiment?

1.2. **Problem 2.** Define a set of events $\mathcal{F}$. You can define this in many ways, but the logical choice would be something similar to what we did in class. At the very least $\mathcal{F}$ should include Ω itself and each outcome contained in Ω.

1.3. **Problem 3.** For your choice of $\mathcal{F}$, define a probability measure, that is, a function

$$\rho : \mathcal{F} \to [0, 1]$$

that is consistent with the Kolmogorov axioms.

1.4. **Problem 4.** Define a random variable on Ω, that is, a function

$$X : \Omega \to \mathbb{R}$$

There are many ways to do this. You should express X as a table of ordered pairs, the first being an element of Ω and the second a real number of your choosing.

1.5. **Problem 5.** State the value of the probability that X assumes each value in its range in this probability space $S = (\Omega, \mathcal{F}, \rho)$, that is,

$$P(X = x) \quad \text{for each} \quad x \in X[\Omega]$$

where $X[\Omega]$ represents the image of Ω under the function X (which is just a precise way of saying all of the values that your random variable can assume).