1. Assignment 9

1.1. **Problem 1.** Suppose X random variables with density function (PDF)

$$f(x) = 1 \quad 0 < x < 1$$

a) Find the density function of the random variable Y defined by

$$Y = X^2$$

b) Find the cumulative distribution function (CDF) of Y

c) Find $E(Y)$ d) Find $V(Y)$

1.2. **Problem 2.** Suppose Y has the *double exponential* distribution

$$f(y) = \frac{1}{2}e^{-|y|} \quad -\infty < y < \infty$$

a) Find the density function (PDF) of the random variable $X = |Y|$

b) Find the cumulative distribution function (CDF) of X

c) Find the expected value $E(X)$

d) Find the variance $V(X)$

1.3. **Problem 3.** Let Z_1 and Z_2 be independent random variables each having the standard normal (i.e., $N(0, 1)$) distribution.

Find the joint density function of the random variables

$$U_1 = Z_1 \quad \text{and} \quad U_2 = Z_1 + Z_2$$

1.4. **Problem 4.** Suppose Y_1 and Y_2 are independent exponentially distributed random variables with common mean β. Show that the joint density of $U_1 = Y_1 + Y_2$ and $U_2 = Y_1/Y_2$ is

$$f_{U_1,U_2}(u_1, u_2) = \begin{cases} \frac{1}{\beta}u_1 e^{-u_1/\beta}\frac{1}{(1+u_2)^2} & 0 < u_1,\ 0 < u_2 \\ 0 & \text{otherwise} \end{cases}$$