## 1. Assignment 8

1.1. Problem 1. Suppose $X$ and $Y$ are random variables with joint probability density function (joint pdf)

$$
f(x, y)=\frac{x^{2}+y^{2}}{26} \quad 0 \leq x \leq 2, \quad 0 \leq y \leq 3
$$

a) Show that $f$ is a valid joint density function (i.e., show that $f(x, y) \geq$ 0 and the integral of $f$ over its support is one)
b) Find the joint cumulative distribution function (CDF) of $X$ and $Y$
c) For any function $u(x, y)$,

$$
E(u)=\int_{0}^{2} \int_{0}^{3} u(x, y) f(x, y) d y d x
$$

Find the expected values $E(X)$ and $E(Y)$ and the variance-covariance matrix of $X$ and $Y$, defined by:

$$
V_{X Y}=\left[\begin{array}{cc}
\sigma_{X}^{2} & \sigma_{X Y} \\
\sigma_{X Y} & \sigma_{Y}^{2}
\end{array}\right]
$$

1.2. Problem 2. Using the joint density function from problem 1),
a) Find the marginal PDFs of $X$ and $Y$ and show that they are valid.
b) Find the marginal CDFs of $X$ and $Y$
c) Find the expected values $E(X)$ and $E(Y)$ and the variances of $X$ and $Y$ using the marginal PDFs. How does your answer compare with 1 c ?
1.3. Problem 3. Using the joint density function from problem 1),
a) Find the conditional PDFs of $X$ given $Y$ and $Y$ given $X$. Show that they are valid.
b) Find the conditional expected value and variance of $X$ given $Y$ and $Y$ given $X$.
c) Find conditional expected value of $X$ given that $Y=1$, and the conditional expected value of $Y$ given that $X=1$.
1.4. Problem 4. Suppose $X$ and $Y$ are random variables having a bivariate Bernoulli distribution with the following probabilities:

| $\mathrm{X}:$ | $\mathrm{Y}: 0$ | 1 |
| :---: | :---: | :---: |
| 0 | $p_{00}$ | $p_{01}$ |
| 1 | $p_{10}$ | $p_{11}$ |

a) Find the marginal density functions of $X$ and $Y$.
b) Find the conditional density functions of $X$ given $Y$ and $Y$ given $X$.
c) Find the expected values of $X$ and $Y$ and their variance-covariance matrix $V_{X Y}$.

