1. Assignment 8

1.1. **Problem 1.** Suppose X and Y are random variables with joint probability density function (joint pdf)

$$f(x,y) = \frac{x^2 + y^2}{26} \quad 0 \le x \le 2, \quad 0 \le y \le 3$$

a) Show that f is a valid joint density function (i.e., show that $f(x, y) \ge 0$ and the integral of f over its support is one)

b) Find the joint cumulative distribution function (CDF) of X and Y

c) For any function u(x, y),

$$E(u) = \int_0^2 \int_0^3 u(x, y) f(x, y) \, dy \, dx$$

Find the expected values E(X) and E(Y) and the variance-covariance matrix of X and Y, defined by:

$$V_{XY} = \left[\begin{array}{cc} \sigma_X^2 & \sigma_{XY} \\ \sigma_{XY} & \sigma_Y^2 \end{array} \right]$$

1.2. **Problem 2.** Using the joint density function from problem 1),

a) Find the marginal PDFs of X and Y and show that they are valid.

b) Find the marginal CDFs of X and Y

c) Find the expected values E(X) and E(Y) and the variances of X and Y using the marginal PDFs. How does your answer compare with 1c?

1.3. **Problem 3.** Using the joint density function from problem 1),

a) Find the conditional PDFs of X given Y and Y given X. Show that they are valid.

b) Find the conditional expected value and variance of X given Y and Y given X.

c) Find conditional expected value of X given that Y = 1, and the conditional expected value of Y given that X = 1.

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1.4. **Problem 4.** Suppose X and Y are random variables having a bivariate Bernoulli distribution with the following probabilities:

$$\begin{array}{c|c|c} X: & Y:0 & 1 \\ \hline 0 & p_{00} & p_{01} \\ 1 & p_{10} & p_{11} \end{array}$$

a) Find the marginal density functions of X and Y.

b) Find the conditional density functions of X given Y and Y given X.

c) Find the expected values of X and Y and their variance-covariance matrix V_{XY} .