## 1. Assignment 7

1.1. **Problem 1.** If a and b are constants and Y is a discrete random variable with moment-generating function  $m_Y(t)$ , show that the moment-generating function of X = aY + b is

$$m_X(t) = e^{tb} m_Y(at)$$

1.2. **Problem 2.** Suppose Y is a random variable with moment-generating function m(t) that is finite for  $|t| \leq b$ . Show that

$$P(X \ge a) \le e^{-at}m(t) \quad \text{for} \quad |t| \le b$$

(Hint: use Markov's inequality)

1.3. **Problem 3.** Let Y be a random variable having a Poisson distribution with parameter  $\lambda$ .

- a) Find E[Y(Y-1)]
- b) Use the results of part a) to show that  $V(Y) = \lambda$ .

1.4. **Problem 4.** Suppose  $Y_n$  is a sequence of random variables such that  $Y_i$  has a binomial B(i,p) distribution for some  $p \in [0,1]$ . Show that for any positive constant  $c \in (0,1)$ ,

$$P(|Y_i - ip| \ge cn) \to 0 \text{ as } n \to \infty$$