1. Assignment 6

1.1. **Problem 1.**

If X and Y are random variables, the **covariance** of X and Y, denoted by σ_{XY} or $\mathrm{Cov}(X,Y)$ is defined as:

$$\sigma_{XY} = \mathrm{Cov}(X,Y) = E(XY) - E(X)\cdot E(Y)$$

where $E(XY)$ is defined as:

$$E(XY) = \sum_x \sum_y xy \cdot P(X = x \text{ and } Y = y)$$

In a binomial experiment with four trials and probability of success p, let Y be the number of successes and X be the number of failures.

a) What are the values of $E(X)$ and $E(Y)$?

b) What is $E(XY)$?

c) What is $\mathrm{Cov}(X,Y)$?

1.2. **Problem 2.**

Suppose (S, Σ, μ) is a probability triple associated with an experiment and Y is a random variable defined on that triple. Show that if we conduct two replications of the experiment in such a way that

$$P(Y_1 = y_1 \text{ and } Y_2 = y_2) = P(Y = y_1)\cdot P(Y = y_2)$$

where Y_1 and Y_2 are the values of the random variable Y associated with the first and second replications, then

$$\mathrm{Cov}(Y_1, Y_2) = 0$$

1.3. **Problem 3.**

Recall that the cumulative distribution function of a random variable Y is defined as:

$$F(y) = P(Y \le y)$$

Suppose a random variable Y associated with an experiment has probability function $p(y)$ and cumulative distribution function $F(y)$. If five independent replications of the experiment are performed, find the cumulative distribution function of the *median* of the five Y values, that is, the third value in the list if the Y values are ordered from largest to smallest. (hint: pick a value y_0 and divide the range of Y into "successes" where $Y_i > y_0$ and "failures" where $Y_i \le y_0$, and try to define the event "the median is less than or equal to y_0 in terms of successes and failures).

1.4. **Problem 4.** Let (S, Σ, μ) be a probability triple and $A, B \in \Sigma$. Define a random variable

$$I_A : S \to \mathbb{R}$$

by:

$$I_A(s) = \begin{cases} 1 & \text{if} \quad s \in A \\ 0 & \text{if} \quad s \notin A \end{cases}$$

and a random vaiable $I_B : S \to \mathbb{R}$:

$$I_B : S \to \mathbb{R}$$

by:

$$I_B(s) = \begin{cases} 1 & \text{if} \quad s \in B \\ 0 & \text{if} \quad s \notin B \end{cases}$$

In terms of elements of Σ and their associated pobability measures,

a) How would you interpret $E(I_A)$?

b) How would you intepret $E(I_A \cdot I_B)$?

c) How would you intepret $E(1 - I_A)$?

d) How would you intepret $E[(1 - I_A)(1 - I_B)]$?