1. Assignment 5

1.1. Problem 1. Suppose Y is a discrete random variable with $E(Y) = \mu$ and a and b are constants. Use the definition of expected value to show that:

a) $E(a \cdot Y) = a \cdot E(Y)$

b) $E(Y + b) = E(Y) + b$

c) $E(a \cdot Y + b) = a \cdot E(Y) + b$

1.2. Problem 2. Suppose Y is a discrete random variable with $V(Y) = \sigma^2$ and a and b are constants. Use the definition of variance to show that:

a) $V(a \cdot Y) = a^2 \cdot V(Y)$

b) $V(Y + b) = V(Y)$

c) $V(a \cdot Y + b) = a^2 \cdot V(Y)$

1.3. Problem 3. Three chips are randomly placed into 3 bins.

a) Describe the probability triple (S, Σ, μ) for this experiment

b) Define a random variable $Y : S \to \mathbb{R}$ such that for $s \in S$, $Y(s)$ is the number of empty bowls. What is $Y[S]$, the range of Y?

c) Assuming all elements of S are equally likely, define a probability function for Y

$$p(y) = P(Y = y) = \mu\left(Y^{-1}[y]\right), \quad y \in Y[S]$$

d) Find $E(Y)$ and $V(Y)$

1.4. Problem 4. Five balls numbered 1 through 5 are placed in an urn and two are selected.

a) Describe the probability triple (S, Σ, μ) for this experiment

b) Define a random variable $Y : S \to \mathbb{R}$ such that for $s \in S$, $Y(s)$ is the *larger* of the two numbers drawn. What is $Y[S]$, the range of Y?

c) Assuming all elements of S are equally likely, define a probability function for Y

$$p(y) = P(Y = y) = \mu\left(Y^{-1}[y]\right), \quad y \in Y[S]$$

d) Find $E(Y)$ and $V(Y)$