# 1. Assignment 3

1.1. **Problem 1.** Consider the problem of placing $r$ indistinguishable tokens into $n$ bins. If the tokens are indistinguishable, a distinct arrangement of tokens into bins is completely described by an $n$-tuple of integers $r_1, r_2, \ldots, r_n$ satisfying

$$r_1 + r_2 + \cdots + r_n = r$$

where $r_i$ is the number of tokens in bin $i$, $1 \leq i \leq n$. Find the number of distinct ways of arranging $r$ tokens in $n$ bins. (hint: represent the $n$ cells as the $n$ spaces between $n+1$ bars. Consider the number of arrangements of $n+1$ bars and $r$ tokens that begin and end with a bar).

1.2. **Problem 2.** With reference to the preceding problem, how many of the distinguishable distributions have no empty bins?

1.3. **Problem 3.** Consider all distinguishable arrangements of two symbols, cirles and stars, containing $r_1$ stars and $r_2$ circles. Assuming $r_1 \geq 2$, How many of these arrangements begin and end with a star?

1.4. **Problem 4.** Define a *run* of stars as a maximal subsequence of stars. For example, in the arrangement

$$\star\star\circ\circ\circ\star\circ\circ\star\star\star\star\circ\circ\star\star\circ$$

there are four runs of stars, one of length one, two of length two, and one of length four. Again considering all distinguishable arrangements of two symbols, cirles and stars, containing $r_1$ stars and $r_2$ circles, how many contain exactly $k$ runs of stars? (hint: use the results of problem 2)