1. ASSIGNMENT 1

1.1. **Problem 1.**

Recall that if A and B are equivalent sets they have the same cardinality, $n(A) = n(B)$. George Cantor extended this concept to infinite sets by defining A and B to be equivalent if there exists a one-to-one correspondence $f : A \to B$. Use Cantor's definition to show that if

$$A = \mathbb{N} = \{1, 2, 3, 4, \ldots\}$$

and

$$B = \{2, 4, 6, 8, \ldots\}$$

then $A \sim B$, that is, A and B have the same number of elements.

1.2. **Problem 2.**

Dirichlet's function $f : \mathbb{R} \to \{0, 1\}$ is defined by

$$f(x) = \begin{cases} 1 & if \quad x \in \mathbb{Q} \; (x \text{ is rational}) \\ 0 & if \quad x \notin \mathbb{Q} \; (x \text{ is irrational}) \end{cases}$$

a) Find $f^{-1}[\{1\}]$

b) Find $f[\mathbb{R} \setminus \mathbb{Q}]$

1.3. **Problem 3.**

Suppose $f : \mathbb{R} \setminus \{-2, 2\} \to \mathbb{R}$ is defined by

$$f(x) = \frac{1}{x^2 - 4}$$

a) Find the range $f[\mathbb{R} \setminus \{-2, 2\}]$

b) Find the image of $(2, \infty)$, $f[(2, \infty)]$

c) Find the inverse image of $\{y \mid 1 \le y \le 3\}$ or $f^{-1}[[1, 3]]$

d) Find the inverse image of $(-\infty, -\frac{1}{4}]$

1.4. **Problem 4.**

Recall from the class notes that if $f : A \to B$ is a function and $E, G \subseteq A$,

$$f[E \cap G] \subseteq f[E] \cap f[G]$$

Find an example of a function $f : A \to B$ and two subsets E and G of its domain for which

$$f[E \cap G] \quad \text{is a proper subset of} \quad f[E] \cap f[G]$$