1. ASSIGNMENT 9

1.1. **Problem 1.** Suppose $E \subset \mathbb{R}$ is a nonempty bounded set and $\sup E \notin E$. Prove that there is a strictly increasing sequence $\{x_n\}$ with $x_n \in E$ for all n such that x_n that converges to $\sup E$.

1.2. **Problem 2.** Suppose that $x_0 \in \mathbb{R}$ and

$$x_n = \frac{1 + x_{n-1}}{2} \quad \text{for} \quad n \in \mathbb{N}$$

Use the monotone convergence theorem to prove that $x_n \to 1$ as $n \to \infty$. (Hint: find the fixed point $\overline{x}$ for this recursion formula and consider the cases $x_0 < \overline{x}$ and $x_0 > \overline{x}$ separately).

1.3. **Problem 3.** If $\{x_n\}$ is a monotone increasing sequence, prove that there is an extended real number x such that $x_n \to x$ as $n \to \infty$.

1.4. **Problem 4.** If the following statement is true, give a proof. If it is false, give a counterexample.

If x_n is a strictly increasing sequence and

$$|x_n| < 1 + \frac{1}{n}, \quad n = 1, 2, \ldots$$

then $x_n \to 1$ as $n \to \infty$.