

1. ASSIGNMENT 8

1.1. **Problem 1.** Use definition 2.14 to prove that the following sequence diverges to ∞ or $-\infty$:

$$x_n = n^2 - n$$

1.2. **Problem 2.** Prove that the following sequence converges to zero:

$$x_n = \frac{\sin(\ln n + n^5 + e^{n^2})}{n}$$

1.3. **Problem 3.** Prove that if x_n is a sequence of positive real numbers that converges to some real number x , then

$$\sqrt{x_n} \rightarrow \sqrt{x} \quad \text{as } n \rightarrow \infty$$

(hint: use (8) of section 1.2.1 for the case $x = 0$)

1.4. **Problem 4.** Prove that for any real number x , there is a sequence r_n of *rational* numbers that converges to x .

You may use Theorem 1.18 (density of rationals). This means, for example, there is a sequence of *rational* numbers whose limit is $\sqrt{2}$.