1. ASSIGNMENT 5

1.1. **Problem 1.** Prove the following statement, or give a counterexample showing it is false: If $E_1, E_2, E_3, \ldots$ is a sequence of finite sets and

$$E := E_1 \times E_2 \times E_3 \times \cdots = \{(x_1, x_2, x_3, \ldots) : x_j \in E_j\}$$

that is, E is the set of all sequences with $x_1 \in E_1$, $x_2 \in E_2$, etc., then E is countable.

1.2. **Problem 2.** Prove that if $x_1 > 2$ and

$$x_{n+1} = 1 + \sqrt{x_n - 1} \quad \text{for all } n \in \mathbb{N}$$

then

$$2 < x_{n+1} < x_n \quad \text{is true for all } n \in \mathbb{N}$$

1.3. **Problem 3.** Prove that $2^n + 3^n$ is a multiple of 5 for every odd $n \in \mathbb{N}$.

1.4. **Problem 4.** Prove that

$$2n + 1 < 2^n, \quad n = 3, 4, 5, \ldots$$