

## 1. ASSIGNMENT 4

1.1. **Problem 1.** Prove the approximation property for infima: If a set  $E \subset \mathbb{R}$  has a finite infimum and  $\epsilon > 0$  is any positive number, then there is a point  $a \in E$  such that

$$\inf E + \epsilon > a \geq \inf E$$

1.2. **Problem 2.** Prove the completeness property for infima: If  $E \subset \mathbb{R}$  is nonempty and bounded below, then  $E$  has a finite infimum.

1.3. **Problem 3.** Prove that the lower bound of a set  $E$  may not be unique, but the infimum  $\inf E$  is unique.

1.4. **Problem 4.** Prove that if  $x$  is an upper bound of  $E \subseteq \mathbb{R}$  and  $x \in E$ , then  $x = \sup E$ .