1. ASSIGNMENT 2

1.1. **Problem 1.**

Define the *positive part* of $a \in \mathbb{R}$ by

$$a^+ := \frac{|a| + a}{2}$$

and the *negative part* of a by

$$a^- := \frac{|a| - a}{2}$$

prove that

$$a^+ = \begin{cases} a & \text{if} \quad a \geq 0 \\ 0 & \text{if} \quad a \leq 0 \end{cases} \qquad \text{and} \quad a^- = \begin{cases} 0 & \text{if} \quad a \geq 0 \\ -a & \text{if} \quad a \leq 0 \end{cases}$$

1.2. **Problem 2.**

Let $a, b \in \mathbb{R}$. Prove that if

$$a > 2 \quad \text{and} \quad b = 1 + \sqrt{a - 1}$$

then $2 < b < a$.

1.3. **Problem 3.**

If $a, b \in \mathbb{R}$, the *arithmetic mean* of a and b is

$$A(a, b) = \frac{a + b}{2}$$

and if $a, b \in [0, \infty)$ the *geometric mean* of a and b is

$$G(a, b) = \sqrt{ab}$$

Prove that if $0 \leq a \leq b$,

$$a \leq G(a, b) \leq A(a, b)$$

1.4. **Problem 4.**

Prove that the sum of a rational number and an irrational number is always irrational.