1. ASSIGNMENT 12

1.1. **Problem 1.** Prove the following statement if it is true, otherwise provide a counterexample:

If

$$\lim_{x\to a^+} f(x) = 0 \quad \text{and} \quad g(x) \geq 1 \quad \forall x \in \mathbb{R} \quad \text{then} \quad \frac{g(x)}{f(x)} \to \infty \text{ as } x \to a^+$$

1.2. **Problem 2.** Prove the following statement if it is true, otherwise provide a counterexample:

If P and Q are polynomials of degree n, there is an $L \in \mathbb{R}$ such that

$$\lim_{x\to\infty} \frac{P(x)}{Q(x)} = \lim_{x\to-\infty} \frac{P(x)}{Q(x)} = L$$

1.3. **Problem 3.** Use the definition of a limit directly (not limit theorems) to prove that the limit

$$\lim_{x\to 0^-} \frac{\sqrt{x^2}}{x}$$

exists. What is the value of the limit?

1.4. **Problem 4.** Prove the following theorem: If

$$f(x) \geq g(x)\ \forall\, x \in \mathbb{R} \quad \text{and} \quad g(x) \to \infty \text{ as } x \to a \quad \text{then} \quad f(x) \to \infty \text{ as } x \to a$$