1. ASSIGNMENT 11

1.1. **Problem 1.** Prove the following theorem:

Suppose $a \in \mathbb{R}$ and f, g, and h are real-valued functions defined everywhere on $I \setminus \{a\}$. Then if

$$f(x) \leq g(x) \leq h(x) \quad \text{for} \quad x \in I \setminus \{a\}$$

and

$$\lim_{x \to a} f(x) \; = \; L \; = \; \lim_{x \to a} h(x)$$

then

$$\lim_{x \to a} g(x) \; = \; L$$

1.2. **Problem 2.** Prove the following theorem:

Suppose $a \in \mathbb{R}$ and f and g are real-valued functions defined everywhere on $I \setminus \{a\}$. Then if

$$f(x) \leq g(x) \quad \text{for} \quad x \in I \setminus \{a\}$$

and

$$\lim_{x \to a} f(x) \quad \text{and} \quad \lim_{x \to a} g(x) \quad \text{exist}$$

then

$$\lim_{x \to a} f(x) \; \leq \; \lim_{x \to a} g(x)$$

1.3. **Problem 3.** Prove that

$$\lim_{x \to 0} \; \sin \frac{1}{x} \quad \text{does not exist}$$

(Hint: You can prove this using a strategy based on the remark at the bottom of Page 70)

1.4. **Problem 4.** Prove the following theorem:

Suppose $a \in \mathbb{R}$ and f and g are real-valued functions defined everywhere on $I \setminus \{a\}$. Then if there is an $M \in \mathbb{R}$ such that

$$|g(x)| \leq M \quad \text{for} \quad x \in I \setminus \{a\}$$

and

$$\lim_{x \to a} f(x) \; = \; 0$$

then

$$\lim_{x \to a} f(x) \cdot g(x) \; = \; 0$$