1. ASSIGNMENT 10

1.1. **Problem 1.**
Suppose $E \subset \mathbb{R}$. A point $a \in \mathbb{R}$ is called a **cluster point** of E if for every $r > 0$,

$$E \cap (a - r, a + r) \quad \text{contains infinitely many points}$$

Prove that every bounded infinite subset of $\mathbb{R}$ has at least one cluster point.

1.2. **Problem 2.**
A subset $E \subset \mathbb{R}$ is said to be **sequentially compact** if and only if every sequence $x_n \in E$ has a convergent subsequence whose limit is in E.

Show that every closed bounded interval $[a, b]$ with $a \leq b$ is sequentially compact.

1.3. **Problem 3.**
a) Prove that there are bounded intervals in $\mathbb{R}$ that are not sequentially compact.

b) Prove that there are closed intervals in $\mathbb{R}$ that are not sequentially compact.

1.4. **Problem 4.**
Suppose x_n and y_n are Cauchy sequences. Without using Theorem 2.29, prove that $x_n + y_n$ is Cauchy.