Discrete Probability Distributions

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A countably infinite set is one that can be put in $1:1$ correspondence with the natural numbers $\mathbb{N} = 1, 2, 3, \ldots$

Discrete Probability Distributions

Of the distributions we have covered, the following have finite sample spaces:

- The Bernoulli distribution: $S = \{0, 1\}$
- The binomial distribution: $S = \{0, 1, 2, \ldots, n\}$

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The following distributions have countably infinite distributions:

- The geometric distribution
- The negative binomial distribution
- The Poisson distribution

In each case, $S = \{0, 1, 2, 3, \ldots\}$

Discrete Probability Distributions

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By way of clarification,

- A **random variable** is a real-valued function defined on the set of outcomes of an experiment.
- A **probability mass function** is a function that maps each value of a random variable into the probability that the random variable assumes that value.

Discrete Probability Distributions

For a Bernoulli random variable, the probability mass function is:

$$p(x) = \begin{cases} p & \text{if} \quad x = 1 \\ 1 - p & \text{if} \quad x = 0 \end{cases}$$

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For a binomial random variable with n trials and probability of success p, denoted $Bin(n, p)$, the pmf is:

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

Discrete Probability Distributions

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For a negative binomial random variable defined as the number of failures preceding the r^{th} success when the probability of success is p, denoted $nb(x; r, p)$, the pmf is:

$$p(x) = \binom{x+r-1}{r-1} p^r (1-p)^x, \quad x = 0, 1, 2, \ldots, n$$

Discrete Probability Distributions

For a Poisson distribution with parameter μ, denoted $p(x; \mu)$, the pmf is:

$$p(x) = \frac{e^{-\mu}\mu^x}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

Discrete Probability Distributions

In all cases the **cumulative distribution function** or CDF, denoted $F(x)$, is defined as the probability that the random variable X assumes a value less than or equal to x:

$$F(x) = P(X \leq x) \; = \; \sum_{i<x} p(x)$$

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The rightmost term is the sum of the values of the probability mass function for all values less than or equal to x.

Discrete Probability Distributions

The **expected value** or mean of a discrete random variable is defined by

$$E(X) = \sum_x x \cdot p(x)$$

which is the sum of x times $p(x)$ over all possible values of x.

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The variance is also denoted by σ^2. The square root of the variance, denoted by σ, is called the **standard deviation**.