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$$\text{Binomial}(n, p) \approx \text{Normal}(np, \sqrt{npq})$$

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The probability density function of a normal random variable is:

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There are a number of drawbacks to tables:

- Usually only values for $\mu = 0$ and $\sigma = 1$ are provided (standard normal)
- Normal variates with other values of μ and σ must be transformed
- Only certain values appear in the table

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