Definitions

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An experiment is a procedure, whether physical or conceptual, with the following properties:

- The experiment is repeatable
- Each time the experiment is performed, exactly one of a set of possible outcomes called the **sample space** occurs

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The probability space consists of three components:

- The **sample space** Ω which is the set of possible outcomes of the experiment
- A collection of subsets of Ω that represent **events**
- A **probability measure** that associates a number between 0 and 1, inclusive with each event

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Some examples of experiments with discrete sample spaces:

- Tossing a coin: $\Omega = \{H, T\}$
- Rolling a die: $\Omega = \{1, 2, 3, 4, 5, 6\}$
- Tossing a coin twice: $\Omega = \{HH, HT, TH, TT\}$
- Tossing a coint until the first heads: $\Omega = \{H, TH, TTH, TTTH, TTTTH, \ldots\}$

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Experiments with continuous sample spaces:

- Choose a number between zero and one, inclusive:
  $\Omega = \{x : 0 \leq x \leq 1\}$
- Choose an individual from a bell curve distribution:
  $\Omega = \{x : -\infty < x < \infty\}$

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If the sample space is discrete, we will use the collection of all possible subsets of ω, which is called the **power set** of ω and denoted (Ω)

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This collection can be described as the set of all intervals of the form $(-\infty, k]$ and their compliments, plus all finite or countable unions of these sets.

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In both cases (discrete and continuous), the collection of subsets $\mathcal{S}$ has the following properties:

- $\mathcal{S}$ includes Ω itself
- If a subset E of Ω belongs to $\mathcal{S}$, so does its compliment
- If each of the subsets E_n, $n = 1, 2, 3, \ldots$ belongs to $\mathcal{S}$, so does their union.

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The technical term for a collection of subsets of this form is a σ-algebra.

Even though the requirement that the set of events be a σ-algebra on the sample space Ω is a key assumption, we will not pay much attention because the two collections we have defined above are both σ-algebras on Ω.

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At the most basic level, every probability experiment will be associated with a probability space having the three components we have defined:

- A sample space Ω
- A collection (S) of subsets of Ω representing events
- A probability measure that assigns a probability value between zero an one to every event

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Every time the experiment is performed, the result is precisely one element of the sample space Ω.

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In the die roll experiment, the compliment of the event $E = \{2, 4\}$ is $E' = \{1, 3, 5, 6\}$