Classical Small Sample (sigma unknow

Recall that the mean of a random sample of size n from a $N(\mu, \sigma)$ population also has a normal distribution:

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More often than not, in practice we cannot assume that we know σ.

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While this seems innocent enough, the situation is much more complicated because s is a random variable while σ is a constant.

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Recall that a sum of the squares of standard normal random variables has a distribution called *Chi-square* with a parameter called the "degrees of freedom" that is the number of independent random variables squared and summed.

As it turns out, s^2/n has a Chi-square distribution with $n-1$ degrees of freedom.

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The t distribution was known for many years as "student's t distribution".

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Directions for obtaining thresholds for the t distribution for two-tailed and one-tailed regions are documented in the technology pages (linked from the web page).

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Note carefully that spreadsheets implement the t distribution functions $TDIST$ and $TINV$ differently from $NORMDIST$ and $NORMINV$.

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The t distribution values can also be obtained using R.

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Numerical values for a t distribution with n degrees of freedom:

$P(X \leq x)$	$pt(x, n)$
x such that $P(X \leq x) = q)$	$qt(x, n)$

Classical Small Sample (sigma unknow

The two-sided $100(1 - \alpha)\%$ confidence interval when both the mean and standard deviation are estimated from the sample is based on:

- the sample mean $\overline{x}$
- the sample standard deviation s
- then sample size n
- the value of α

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The lower and upper limits of the two-sided confidence interval are:

Lower limit	$\overline{x} + qt(\alpha/2, n-1) \cdot s/\sqrt{n}$
Upper limit	$\overline{x} + qt(1-\alpha/2, n-1) \cdot s/\sqrt{n}$

Classical Small Sample (sigma unknow

The upper one-sided $100(1-\alpha)\%$ confidence interval when both the mean and standard deviation are estimated from the sample is:

Lower limit	$-\infty$
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The lower one-sided $100(1-\alpha)\%$ confidence interval when both the mean and standard deviation are estimated from the sample is:

Lower limit	$\overline{x} + qt(\alpha, n-1) \cdot s/\sqrt{n}$
Upper limit	∞

Example

A random sample of 100 students have their heights measured. If the average score is 69.3 inches with a sample standard deviation of 3.4, construct a two-sided 95% confidence interval for the mean height of students.

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In this case, $\alpha = .05$, $n = 100$, $\overline{x} = 69.3$ and $s = 3.4$. Since we have no reason to assume we know the standard deviation of heights, we will use a t distribution to construct the confidence interval.

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In this case, $\alpha = .05$, $n = 100$, $\overline{x} = 69.3$ and $s = 3.4$. Since we have no reason to assume we know the standard deviation of heights, we will use a t distribution to construct the confidence interval.

Lower limit	$\overline{x} + qt(\alpha/2, n-1) \cdot s/\sqrt{n}$
Lower limit	$69.3 + qt(.025, 99) * 3.4/10$=$68.625$
Upper limit	$\overline{x} + qt(1-\alpha/2, n-1) \cdot s/\sqrt{n}$
Upper limit	$69.3 + qt(0.975, 99) * 3.4/10$=$69.975$

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Example

The speed of 250 cars in the right hand lane of an interstate highway is measured with radar. If the average speed is $60.2mph$ and the sample standard deviation is 5.3, construct a 99% confidence interval for the mean vehicle speed.

In this case, $\alpha = .01$, $n = 250$, $\overline{x} = 60.2$ and $s = 5.3$. Since we have no reason to assume we know the true standard deviation of speeds, we will use a t distribution to construct the confidence interval.

Lower limit	$\overline{x} + qt(\alpha/2, n-1) \cdot s/\sqrt{n}$
Lower limit	$60.2 + qt(.005, 249) * 5.3/\sqrt{250}$=$59.330$
Upper limit	$\overline{x} + qt(1-\alpha/2, n-1) \cdot s/\sqrt{n}$
Upper limit	$60.2 + qt(0.995, 99) * 5.3/\sqrt{250}$=$61.070$

Example - Technology Page

Viscosity of oil from 50 test vehicles after $10,000$ miles of driving averaged 43.1 with a standard deviation of 10.3. Find a two-sided 95% confidence interval for the mean viscosity of oil after $10,000$ miles.

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Download the spreadsheet from the Technology Page in the section:

- Confidence Intervals for Population Means
- Sigma unknown

Example - Technology Page

A sample of 80 lobsters caught in Narragansett Bay averaged 12.4 ounces in weight with a standard deviation of 3.1. Find a two-sided 99% confidence interval for the mean weight of lobsters in Narragansett Bay.

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Example - R Simulations

There is an R program that simulates the behavior of classical small sample confidence intervals from the notes and handouts section of the web page. The link is named:

Classical small sample confidence intervals (R simulation)

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It should be clear from the results that when the sample size reaches about 30, and especially when it reaches 50, there is very little error in "incorrectly" using the large sample method.