Expected Value of a Discrete Random V

The **expected value** of a discrete random variable X is given by:

$$E(X) = \sum_{Range(X)} x \cdot p(x)$$

where:

- $Range(X)$ is the range of the random variable X, that is, the set of values X assumes.
- $p(x)$ is the probability mass function of X, $p(x) = P(X = x)$

Expected Value of a Bernoulli Random

The expected value of a **Bernoulli** random variable X with probability of success p is given by:

$$E(X) = \sum_{x=0}^{1} x \cdot p(x)$$

where:

- $p(1) = p$
- $p(0) = 1 - p$

so

$$E(X) \;=\; 1 \cdot p(1) + 0 \cdot p(0) \;=\; 1 \cdot p + 0 \cdot (1 - p)$$

So, the expected value of a Bernoulli random variable is the same as the probability of success.

Expected Value of a Binomial Random

The expected value of a **binomial** random variable X with n trials and probability of success p is given by:

$$E(X) = \sum_{x=0}^{n} x \cdot \binom{n}{x} p^x (1-p)^{n-x}$$

where:

- n is the number of trials in the binomial experiment
- p is the probability of success for each of the n Bernoulli trials in the binomial experiment.

Expected Value of a Binomial Random

With a bit of clever algebra it can be shown that

$$E(X) \;=\; \sum_{x=0}^{n} x \cdot \binom{n}{x} p^x (1-p)^{n-x} \;=\; np$$

So, the expected value of a binomial random variable is the product of the number of trials and the probability of success.

Expected Value of a Geometric Random

The expected value of a **geometric** random variable X with probability of success p is given by:

$$E(X) = \sum_{x=0}^{\infty} x \cdot p(1-p)^x$$

where p is the probability of success in the Bernoulli trials comprising the geometric experiment.

Expected Value of a Geometric Random

The expected value of a **geometric** random variable X with probability of success p is given by:

$$E(X) = \sum_{x=0}^{\infty} x \cdot p(1-p)^x$$

where p is the probability of success in the Bernoulli trials comprising the geometric experiment.

From the results for sums of geometric series,

$$E(X) = \sum_{x=0}^{\infty} x \cdot p(1-p)^x \; = \; \frac{1-p}{p}$$

Expected Value of a Negative Binomial

The expected value of a **negative binomial** random variable X with r successes and probability of success p is given by:

$$E(X) = \sum_{x=0}^{\infty} x \cdot \binom{x+r-1}{x} p^r (1-p)^x$$

where p is the probability of success in the Bernoulli trials comprising the negative experiment and r is the number of successes required to stop.

Expected Value of a Negative Binomial

The expected value of a **negative binomial** random variable X with r successes and probability of success p is given by:

$$E(X) = \sum_{x=0}^{\infty} x \cdot \binom{x+r-1}{x} p^r (1-p)^x$$

where p is the probability of success in the Bernoulli trials comprising the negative experiment and r is the number of successes required to stop.

From the results for sums of power series,

$$E(X) = \sum_{x=0}^{\infty} x \cdot \binom{x+r-1}{x} p^r (1-p)^x \; = \; \frac{r(1-p)}{p}$$

Expected Value of a Poisson Random V

The expected value of a **Poisson** random variable X with parameter λ is given by:

$$E(X) = \sum_{x=0}^{\infty} x \cdot \frac{\lambda^x e^{-\lambda}}{x!}$$

where λ is the (constant) value of np for the sequence binomial distributions whose limit is the Poisson.

Expected Value of a Poisson Random V

The expected value of a **Poisson** random variable X with parameter λ is given by:

$$E(X) = \sum_{x=0}^{\infty} x \cdot \frac{\lambda^x e^{-\lambda}}{x!}$$

where λ is the (constant) value of np for the sequence binomial distributions whose limit is the Poisson.

From the results for sums of power series,

$$E(X) = \sum_{x=0}^{\infty} x \cdot \frac{\lambda^x e^{-\lambda}}{x!} \; = \; \lambda$$