Expected Value of a Discrete Random V

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Recall that a **random variable** is a function that maps the sample space of an experiment into the real numbers:

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The **expected value** of the random variable X is defined by:

$$E(X) = \sum_{Range(X)} x \cdot p(x)$$

where $p(x)$ is the probability mass function.

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For a discrete random variable, the probability mass function maps each value that the random variable can take into the probability that it takes that value.

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Example: Suppose random variable X represents the number of successes in a binomial experiment with three trials and probability of success 0.4 on each trial. The probability mass function, in tabular form, is:

x	R code for $p(x)$	$p(x)$
0	`dbinom(0,3,0.4)`	0.216
1	`dbinom(1,3,0.4)`	0.432
2	`dbinom(2,3,0.4)`	0.288
3	`dbinom(3,3,0.4)`	0.064

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The **expected value** of the random variable X is:

$$E(X) = \sum_{k=0}^{3} k \cdot p(k) = 0 \cdot 0.216 + 1 \cdot 0.432 + 2 \cdot 0.288 + 3 \cdot 0.064$$

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With a bit of algebra one can show that the expected value $E(X)$ of a binomial random variable representing an experiment with n trials and probability of success p is always:

$$E(X) = np$$

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If we repeat the experiment many times, the **average** number of successes will be approach 1.2 as the number of replications of the experiment grows.

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An interpretation we can give is the following:

If we repeat the experiment many times, the **average** number of successes will be approach 1.2 as the number of replications of the experiment grows.

This statement represents what is known as the **law of large numbers**

Expected Value of a Discrete Random V

More precisely, if the random variable X represents a probability experiment, and we replicate the experiment n times, as n becomes large,

$$\frac{1}{n}\sum_{i=1}^{n} X_i \;\rightarrow\; E(X) \quad \text{with probability } 1$$

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Essentially it says that, while we cannot hope to predict the outcome of a single experiment, if we can replicate the experiment many times, we **can** predict the long term average behavior with great certainty.

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