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Expected value and variance:

$$E(X) \;=\; p \qquad\qquad V(X) \;=\; p(1 - p)$$

The Binomial Distribution

n Bernoulli trials with common parameter p

$$\Omega \;=\; \{\text{all sequences of n letters, each S or F}\}$$

$$X : \Omega \to \{0, 1, \ldots, n\} \quad X \text{ is the number of successes}$$

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Parameters: n is the number of trials, p is the probability of success.

Probability mass function:

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

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Probability mass function:

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n$$

Expected value and variance:

$$E(X) \ = \ np \qquad V(X) \ = \ np(1-p)$$

The Geometric Distribution

Conduct Bernoulli trials until the first success.

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Expected value and variance:

$$E(X) \;=\; \frac{1-p}{p} \qquad V(X) \;=\; \frac{1-p}{p^2}$$

The Negative Binomial Distribution

Conduct Bernoulli trials until the r^{th} success.

$$\Omega \;=\; \text{sequences with r S's ending with an S}$$

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Probability mass function:

$$p(x) \; = \; \binom{x+r-1}{x} p^r (1-p)^x, \quad x = 0, 1, 2, \ldots$$

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Expected value and variance:

$$E(X) \; = \; \frac{r(1-p)}{p} \qquad V(X) \; = \; \frac{r(1-p)}{p^2}$$

The Poisson Distribution

Limiting form of binomial distribution.

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Parameters: λ (fixed value of np in limiting binomial distribution)

Probability mass function:

$$p(x) = \frac{\lambda^x e^{-\lambda}}{x!}, \quad x = 0, 1, 2, 3, \ldots$$

Expected value and variance:

$$E(X) = \lambda \qquad V(X) = \lambda$$