The Uniform Distribution

Functions

- spreadsheet: `RAND()`
- R: `runif(1)`
- BUGS: `x∼ dunif(0,1)`

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- Expected value: $E(X) = 1/2$
- Variance: $V(X) = 1/12$

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The uniform is one of the simplest continuous distributions, and the starting point for most simulation procedures.

The Normal Distribution

Functions:

- spreadsheet: `NORMINV(RAND(),mu,sigma)`
- R: `rnorm(1,mu,sigma)`
- BUGS: `x`$\sim$ `dnorm(mu,tau)` with tau=1/sigma

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The normal is the most important distribution in statistics because of a number of theoretical results including the **central limit theorem**.

The Chi-Square Distribution

Functions:

- spreadsheet: `CHIINV(RAND(),k)`
- R: `rchisq(1,k)`
- BUGS: `x`$\sim$ `dchisq(k)`

The Chi-Square Distribution

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- Variance: $V(X) = 2k$

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- Variance: $V(X) = 2k$

Many "sums of squares" that appear in statistics have a chi-square distribution when the underlying population is normal.

The t Distribution

Functions:

- spreadsheet: `TINV(RAND(),k)`
- R: `rt(1,k)`
- BUGS: `x`$\sim$ `dt(0,1,k)`

The t Distribution

Functions:

- spreadsheet: `TINV(RAND(),k)`
- R: `rt(1,k)`
- BUGS: `x`$\sim$ `dt(0,1,k)`

- Support: $(-\infty, \infty)$
- Expected value: $E(X) = 0$
- Variance: $V(X) = k/(k-2)$ for $k > 2$

The t Distribution

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- Expected value: $E(X) = 0$
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The t distribution becomes almost indistinguishable from the normal as the number of degrees of freedom becomes larger. For k above 30 there is not much difference between them.

The F Distribution

Functions:

- spreadsheet: `FINV(RAND(),`k_1`,`k_2`)`
- R: `rf(1,`k_1`,`k_2`)`

The ratio of two independent chi-square random variables divided by their respective degrees of freedom has an F distribution.

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The ratio of two independent chi-square random variables divided by their respective degrees of freedom has an F distribution.

- support: $[0, \infty)$
- parameters: degrees of freedom (numerator and denominator) k_1, k_2
- expected value $E(X)$: $k_2/(k_2 - 2)$ for $k_2 > 2$
- variance $V(X)$: (complicated)

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Used in many common hypothesis tests

The Exponential Distribution

Functions:

- R: `rexp(1,lambda)`
- BUGS: `x~ dexp(lambda)`

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- R: `rexp(1,lambda)`
- BUGS: `x`$\sim$ `dexp(lambda)`

- support: $[0, \infty)$
- parameters: λ
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- variance $V(X)$: $1/\lambda^2$

Plays an important role in time-to-failure models and queueing theory

The Beta Distribution

Functions:

- R: `rbeta(1,a,b)`
- BUGS: `p`$\sim$ `dbeta(a,b)`

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- support: $[0, 1]$
- parameters: a, b
- expected value $E(X)$: $a/(a + b)$
- variance $V(X)$: $ab/((a + b)^2(a + b + 1))$

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Commonly used as a prior distribution in Bayesian analysis

The Gamma Distribution

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- R: `rgamma(1,a,b)`
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- BUGS: `x`$\sim$ `dgamma(a,b)`

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- parameters: a, b
- expected value $E(X)$: a/b
- variance $V(X)$: a/b^2

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- support: $[0, \infty)$
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Commonly used as a prior distribution in Bayesian analysis

The Logistic Distribution

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- parameters: μ, s (location and scale)
- expected value $E(X)$: μ
- variance $V(X)$: $s^2\pi^2/3$

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- support: $(-\infty, \infty)$
- parameters: μ, s (location and scale)
- expected value $E(X)$: μ
- variance $V(X)$: $s^2\pi^2/3$

Commonly used for the response in logistic regression

The Pareto Distribution

Functions:

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- BUGS: `x`$\sim$ `dpar(`α`,k)`

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Functions:

- R: `rlogis(1,`α`,k)`
- BUGS: `x`$\sim$ `dpar(`α`,k)`

- support: $[k, \infty)$
- parameters: α, k (location and shape)
- expected value $E(X)$: $\alpha k/(\alpha - 1)$ ($\alpha > 1$)
- variance $V(X)$: $\alpha k^2/((\alpha - 1)^2(\alpha - 2))$ ($\alpha > 2$)

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Commonly used for extreme value distributions

The Weibull Distribution

Functions:

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- R: `rweibull(1,`v, λ`)`
- BUGS: `x`$\sim$ `dweib(`v, λ`)`

- support: $[0, \infty)$
- parameters: v, λ (shape and scale)
- expected value $E(X)$: $v\Gamma(1 + 1/k)$
- variance $V(X)$: $v^2(\Gamma(1 + 2/k) - (\Gamma(1 + 1/k))^2)$

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- expected value $E(X)$: $v\Gamma(1 + 1/k)$
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Commonly used for time to failure applications with long wait times