General Definition

For any two events A and B with $P(B) > 0$, the **conditional probability** of A given that B has occurred is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

General Definition

For any two events A and B with $P(B) > 0$, the **conditional probability** of A given that B has occurred is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Alternatively one can write this as

$$P(A \cap B) = P(A|B) \cdot P(B)$$

General Definition

For any two events A and B with $P(B) > 0$, the **conditional probability** of A given that B has occurred is:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Alternatively one can write this as

$$P(A \cap B) = P(A|B) \cdot P(B)$$

The later is called the *multiplication rule* for $P(A \cap B)$.

General Definition

The multiplication rule extends to more than two events:

$$P(A \cap B \cap C) = P(C|A \cap B) \cdot P(A|B) \cdot P(B)$$

General Definition

The multiplication rule extends to more than two events:

$$P(A \cap B \cap C) = P(C|A \cap B) \cdot P(A|B) \cdot P(B)$$

In fact this can be continued indefinitely.

General Definition

The multiplication rule extends to more than two events:

$$P(A \cap B \cap C) = P(C|A \cap B) \cdot P(A|B) \cdot P(B)$$

In fact this can be continued indefinitely.

This result is useful for building up probibility trees of complicated sequences of events.

The Law of Total Probability

Suppose $A_1, \ldots, A_k$ are mutually exclusive events, one of which must occur. Then for any event B,

$$P(B) = P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + \cdots + P(B|A_k) \cdot P(A_k)$$

The Law of Total Probability

Suppose $A_1, \ldots, A_k$ are mutually exclusive events, one of which must occur. Then for any event B,

$$P(B) = P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + \cdots + P(B|A_k) \cdot P(A_k)$$

Suppose the Red Sox have clinched the pennant, but the national league championship game between Chicago and Pittsburgh has yet to be played. Let B be the event that Boston wins the world series, A_1 be the event that Chicago wins the LCS, and A_2 be the event that Pittsburgh wins the LCS.

The Law of Total Probability

Suppose $A_1, \ldots, A_k$ are mutually exclusive events, one of which must occur. Then for any event B,

$$P(B) = P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2) + \cdots + P(B|A_k) \cdot P(A_k)$$

Suppose the Red Sox have clinched the pennant, but the national league championship game between Chicago and Pittsburgh has yet to be played. Let B be the event that Boston wins the world series, A_1 be the event that Chicago wins the LCS, and A_2 be the event that Pittsburgh wins the LCS.

The probability that Boston wins the world series is:

$$P(B) = P(B|A_1) \cdot P(A_1) + P(B|A_2) \cdot P(A_2)$$

Baye's Theorem

Suppose $A_1, \ldots, A_k$ are mutually exclusive events, one of which must occur, with *prior* probabilities A_i. Then for any event B, the *posterior* probability of A_j given that B has occurred is:

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)}$$

Baye's Theorem

Suppose $A_1, \ldots, A_k$ are mutually exclusive events, one of which must occur, with *prior* probabilities A_i. Then for any event B, the *posterior* probability of A_j given that B has occurred is:

$$P(A_j|B) = \frac{P(A_j \cap B)}{P(B)}$$

Using the law of total probability, this can be written as

$$\frac{P(B|A_j) \cdot P(A_j)}{\sum_{i=1}^{k} P(B|A_i) \cdot P(A_i)}, \quad j = 1, \ldots, k$$

Independence

Two events A and B are said to be **independent** if:

$$P(A|B) = P(A)$$

Independence

Two events A and B are said to be **independent** if:

$$P(A|B) = P(A)$$

That is to say, knowledge that B has occurred gives us no information about whether or not A will occur.

Independence

Two events A and B are said to be **independent** if:

$$P(A|B) = P(A)$$

That is to say, knowledge that B has occurred gives us no information about whether or not A will occur.

Events that are not independent are said to be **dependent**

Independence

Two events A and B are independent if and only if:

$$P(A \cap B) = P(A) \cdot P(B)$$

Independence

Two events A and B are independent if and only if:

$$P(A \cap B) = P(A) \cdot P(B)$$

In other words, A and B are independent if and only if the probability of both A and B occurring is the product of the probabilities that each individual event occurs.

Independence

Two events A and B are independent if and only if:

$$P(A \cap B) = P(A) \cdot P(B)$$

In other words, A and B are independent if and only if the probability of both A and B occurring is the product of the probabilities that each individual event occurs.

Events $A_1, A_2, \ldots, A_n$ are called *mutually independent* if for every possible subset of these n events, the probability that each event in the subset occurs is the product of the individual probabilities.