Bell Curves for Sample Proportions

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This means we can use the NORMDIST function exactly as we did for populations and sample means.

Example

The proportion of the human population that has Type A blood is 40 percent.

If a sample of 75 individuals is taken, the distribution of the sample proportion $\overline{x}$,

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$$\frac{\text{Number of Individuals with Type A Blood}}{\text{Number of Individuals in Sample}} = \frac{x}{n}$$

The sample proportion $\overline{x}$ has a bell curve distribution (approximately) with:

$$\text{mean} = 0.4 \quad \text{standard deviation} = \sqrt{\frac{(0.4)(1-0.4)}{75}} = .0566$$

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If we have 33 or fewer individuals with Type A blood, the sample proportion $\overline{x}$ must be at most:

$$\overline{x} = \frac{33}{75} = .44$$

So we want the proportion of a bell curve population with a mean of 0.40 and a standard deviation of 0.0566 that lies below 0.44, which is:

=NORMDIST(0.44,0.40,0.0566,true)=0.76

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- The sample size is n
- We want the proportion of samples with x or fewer individuals that have the characteristic

In this case, the proportion of samples that will have x or fewer individuals with the characteristic is:

=NORMDIST(x/n,p,SQRT(p*(1-p)/n),true)

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In this case,

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- The sample size n is 100
- The number of individuals in the sample with Type O blood is $x = 40$

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In human populations, the proportion with Type O blood is 45 percent.

Find the proportion of samples of size 100 that contain 40 or fewer individuals with Type O blood.

In this case,

- The population proportion p is 0.45
- The sample size n is 100
- The number of individuals in the sample with Type O blood is $x = 40$

Then the proportion is given by the formula:

=NORMDIST(40/100,0.45,SQRT(0.4*(1-0.4)/100),true)