Bernoulli Random Variables

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In general we can consider a Bernoulli random variable to be the result of an experiment with two outcomes, which for convenience we will label "success" and "failure"

As before we define the Bernoulli random variable X by agreeing to assign the value of 1 to X if the result of the experiment is "success", and zero if the result is "failure":

$$X = \begin{cases} 1 & \text{if the outcome of the experiment is "success"} \\ 0 & \text{if the outcome of the experiment is "failure"} \end{cases}$$

Discrete Distributions

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Another way to say this is that we take binomial random variables with larger and larger n, but we keep the *expected number of successes* $np = l$ the same for all of them.

The limit of the distribution of such a sequence of random variables as $n \to \infty$ is a Poisson.

The Poisson Distribution

The mean or expected value of a Poisson random variable with parameter l is:

$$\mu = E(X) = l$$

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The variance of a Poisson random variable with parameter l is:

$$\sigma^2 = V(X) \ = \ l$$

The Poisson Distribution

Now we will perform some numerical experiments.

First generate a sample of $1,000,000$ observations for a Poisson experiment with mean 4:

```
x<-rpois(1000000,4)
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hist(x)
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```

The results through $X = 6$ should look something like:

0	1	2	3	4	5	
18371	73359	146588	195040	195902	155639	10409

The Poisson Distribution

0	1	2	3	4	5	
18371	73359	146588	195040	195902	155639	10409

Now compare the frequencies to the probabilities.

First compute the probability that $X = 0$:

```
dpois(0,4)
```

The Poisson Distribution

0	1	2	3	4	5	
18371	73359	146588	195040	195902	155639	10409

Now compare the frequencies to the probabilities.

First compute the probability that $X = 0$:

```
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The result should be something like

```
[1] 0.01831564
```

The Poisson Distribution

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Now compare the frequencies to the probabilities.

First compute the probability that $X = 0$:

```
dpois(0,4)
```

The result should be something like

```
[1]  0.01831564
```

To get the probability that $X = 1$ enter

```
dpois(1,4)
```

The Poisson Distribution

0	1	2	3	4	5	
18371	73359	146588	195040	195902	155639	10409

Now compare the frequencies to the probabilities.

First compute the probability that $X = 0$:

```
dpois(0,4)
```

The result should be something like

```
[1]  0.01831564
```

To get the probability that $X = 1$ enter

```
dpois(1,4)
```

This time the results should look something like:

```
[1]  0.07326256
```

The Poisson Distribution

0	1	2	3	4	5	
18371	73359	146588	195040	195902	155639	10409

Next compute the probability that $X = 3$:

```
dpois(3,4)
```

The Poisson Distribution

0	1	2	3	4	5	
18371	73359	146588	195040	195902	155639	10409

Next compute the probability that $X = 3$:

```
dpois(3,4)
```

The result should be something like

```
[1]  0.1953668
```

The Poisson Distribution

0	1	2	3	4	5	
18371	73359	146588	195040	195902	155639	10409

Next compute the probability that $X = 3$:

```
dpois(3,4)
```

The result should be something like

```
[1] 0.1953668
```

To get the probability that $X = 5$ enter

```
dpois(5,4)
```

The Poisson Distribution

0	1	2	3	4	5	
18371	73359	146588	195040	195902	155639	10409

Next compute the probability that $X = 3$:

```
dpois(3,4)
```

The result should be something like

```
[1]  0.1953668
```

To get the probability that $X = 5$ enter

```
dpois(5,4)
```

This time the results should look something like:

```
[1]  0.1562935
```

The Poisson Distribution

The expected value $E(X)$ in this case is:

$$E(X) \ = l \ = \ 4$$

The Poisson Distribution

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To compute the sample mean $\overline{x}$, enter

```
mean(x)
```

The Poisson Distribution

The expected value $E(X)$ in this case is:

$$E(X) = l = 4$$

To compute the sample mean $\overline{x}$, enter

`mean(x)` The result should be something like

```
[1]  4.000327
```

The Poisson Distribution

The variance $V(X)$ in this case is:

$$V(X) = l = 4$$

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To compute the sample variance s^2, enter

```
var(x)
```

The Poisson Distribution

The variance $V(X)$ in this case is:

$$V(X) = l = 4$$

To compute the sample variance s^2, enter

`var(x)` The result should be something like

```
[1] 4.009417
```

The Poisson Distribution

The number of cars arriving at a toll booth per minute has a Poisson distribution with $l = 4.6$.

Find the probability that in a given minute exactly 4 cars arrive at the toll booth.

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Solution: 0.1875277

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Find the probability that in a given minute exactly 4 cars arrive at the toll booth.

Solution: 0.1875277

```
dpois(4,4.6)
```

The Poisson Distribution

The number of cars arriving at a toll booth per minute has a Poisson distribution with $l = 4.6$.

Find the probability that in a given minute 4 or fewer cars arrive at the toll booth.

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Find the probability that in a given minute 4 or fewer cars arrive at the toll booth.

Solution: 0.513234

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Find the probability that in a given minute 4 or fewer cars arrive at the toll booth.

Solution: 0.513234

```
ppois(4,4.6)
```

The Poisson Distribution

The number of cars arriving at a toll booth per minute has a Poisson distribution with $l = 4.6$.

Find the probability that in a given minute more than 8 cars arrive at the toll booth.

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Solution: 0.04507196

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Find the probability that in a given minute more than 8 cars arrive at the toll booth.

Solution: 0.04507196

```
1-ppois(8,4.6)
```

The Poisson Distribution

The number of cars arriving at a toll booth per minute has a Poisson distribution with $l = 4.6$.

Find the probability that in a given minute between 3 and 5 cars arrive at the toll booth.

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Find the probability that in a given minute between 3 and 5 cars arrive at the toll booth.

Solution: 0.5231208

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The number of cars arriving at a toll booth per minute has a Poisson distribution with $l = 4.6$.

Find the probability that in a given minute between 3 and 5 cars arrive at the toll booth.

Solution: 0.5231208

```
ppois(5,4.6)-ppois(3-1,4.6)
```

The Poisson Distribution

The number of deer ticks in a square yard of forest floor has a Poisson distribution with a mean of 12.

Find the probability that a randomly chosen square yard contains more than 10 deer ticks.

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Solution: 0.7576078

The Poisson Distribution

The number of deer ticks in a square yard of forest floor has a Poisson distribution with a mean of 12.

Find the probability that a randomly chosen square yard contains more than 10 deer ticks.

Solution: 0.7576078

```
1-ppois(10-1,12)
```