Bernoulli Random Variables

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Bernoulli Random Variables

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In general we can consider a Bernoulli random variable to be the result of an experiment with two outcomes, which for convenience we will label "success" and "failure"

As before we define the Bernoulli random variable X by agreeing to assign the value of 1 to X if the result of the experiment is "success", and zero if the result is "failure":

$$X = \begin{cases} 1 & \text{if the outcome of the experiment is "success"} \\ 0 & \text{if the outcome of the experiment is "failure"} \end{cases}$$

Discrete Distributions

Now consider a series of *independent* experiments, each of which produces a Bernoulli random variable with probability of success p (p is the same for all of the trials)

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The following discrete probability distributions arise from this model:

If the number of trials n is fixed in advance, the number of successes X has a **binomial** distribution

If trials continue indefinitely until the first success is obtained, the number of failures obtained X has a **geometric** distribution.

If trials continue indefinitely until the r^{th} success is obtained, the number of failures obtained X has a **negative binomial** distribution.

The Negative Binomial Distribution

The negative binomial experiment with parameters r and p consists of:

- Independent Bernoulli trials are performed until r "successes" are obtained
- The random variable X is the number of failures obtained
- The probability of success p is the same for all trials

The negative binomial Distribution

The mean or expected value of a negative binomial random variable is:

$$\mu = E(X) = \frac{r(1-p)}{p}$$

The negative binomial Distribution

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$$\mu = E(X) = \frac{r(1-p)}{p}$$

The variance of a negative binomial random variable is:

$$\sigma^2 = V(X) \ = \ \frac{f(1-p)}{p^2}$$

The negative binomial Distribution

Now we will perform some numerical experiments.

First generate a sample of $1,000,000$ observations for a negative binomial experiment with $r = 3$ and probability of success $p = 0.4$ at each trial:

```
x<-rnbinom(1000000,3,0.4)
```

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To get a table of the results enter

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To get a table of the results enter

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```

The results through $X = 6$ should look something like:

0	1	2	3	4	5	
63851	114835	138084	138075	124924	103961	8355

The Negative Binomial Distribution

0	1	2	3	4	5	
63851	114835	138084	138075	124924	103961	8355

Now compare the frequencies to the probabilities.

First compute the probability that $X = 0$:

```
dnbinom(0,3,0.4)
```

The Negative Binomial Distribution

0	1	2	3	4	5	
63851	114835	138084	138075	124924	103961	8355

Now compare the frequencies to the probabilities.

First compute the probability that $X = 0$:

```
dnbinom(0,3,0.4)
```

The result should be something like

```
[1]  0.064
```

The Negative Binomial Distribution

0	1	2	3	4	5	
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Now compare the frequencies to the probabilities.

First compute the probability that $X = 0$:

```
dnbinom(0,3,0.4)
```

The result should be something like

```
[1]  0.064
```

To get the probability that $X = 1$ enter

```
dnbinom(1,3,0.4)
```

The Negative Binomial Distribution

0	1	2	3	4	5	
63851	114835	138084	138075	124924	103961	8355

Now compare the frequencies to the probabilities.

First compute the probability that $X = 0$:

```
dnbinom(0,3,0.4)
```

The result should be something like

```
[1]  0.064
```

To get the probability that $X = 1$ enter

```
dnbinom(1,3,0.4)
```

This time the results should look something like:

```
[1]  0.1152
```

The Negative Binomial Distribution

0	1	2	3	4	5	
63851	114835	138084	138075	124924	103961	8355

Next compute the probability that $X = 2$:

```
dnbinom(2,3,0.4)
```

The Negative Binomial Distribution

0	1	2	3	4	5	
63851	114835	138084	138075	124924	103961	8355

Next compute the probability that $X = 2$:

```
dnbinom(2,3,0.4)
```

The result should be something like

```
[1] 0.13284
```

The Negative Binomial Distribution

0	1	2	3	4	5	
63851	114835	138084	138075	124924	103961	8355

Next compute the probability that $X = 2$:

```
dnbinom(2,3,0.4)
```

The result should be something like

```
[1]  0.13284
```

To get the probability that $X = 5$ enter

```
dnbinom(5,3,0.4)
```

The Negative Binomial Distribution

0	1	2	3	4	5	
63851	114835	138084	138075	124924	103961	8355

Next compute the probability that $X = 2$:

```
dnbinom(2,3,0.4)
```

The result should be something like

```
[1]  0.13284
```

To get the probability that $X = 5$ enter

```
dnbinom(5,3,0.4)
```

This time the results should look something like:

```
[1]  0.1045094
```

The Negative Binomial Distribution

The expected value $E(X)$ in this case is:

$$E(X) = \frac{r(1-p)}{p} = \frac{3(.6)}{.4} = 4.5$$

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To compute the sample mean $\overline{x}$, enter

```
mean(x)
```

The Negative Binomial Distribution

The expected value $E(X)$ in this case is:

$$E(X) = \frac{r(1-p)}{p} = \frac{3(.6)}{.4} = 4.5$$

To compute the sample mean $\overline{x}$, enter

`mean(x)` The result should be something like

```
[1] 4.50794
```

The Negative Binomial Distribution

The variance $V(X)$ in this case is:

$$V(X) = \frac{r(1-p)}{p^2} = \frac{3(.6)}{.4^2} = 11.25$$

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To compute the sample variance s^2, enter

```
var(x)
```

The Negative Binomial Distribution

The variance $V(X)$ in this case is:

$$V(X) = \frac{r(1-p)}{p^2} = \frac{3(.6)}{.4^2} = 11.25$$

To compute the sample variance s^2, enter

`var(x)` The result should be something like

```
[1] 11.23359
```

The Negative Binomial Distribution

A fair coin is tossed until the third heads comes up.

Find the probability that the third heads comes up on the fifth toss.

The Negative Binomial Distribution

A fair coin is tossed until the third heads comes up.

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Solution: 0.1875

The Negative Binomial Distribution

A fair coin is tossed until the third heads comes up.

Find the probability that the third heads comes up on the fifth toss.

Solution: 0.1875

```
dnbinom(5-3,3,0.5)
```

The Negative Binomial Distribution

A fair coin is tossed until the third heads comes up.

Find the probability that the third heads comes up on the fifth toss or sooner.

The Negative Binomial Distribution

A fair coin is tossed until the third heads comes up.

Find the probability that the third heads comes up on the fifth toss or sooner.

Solution: 0.5

The Negative Binomial Distribution

A fair coin is tossed until the third heads comes up.

Find the probability that the third heads comes up on the fifth toss or sooner.

Solution: 0.5

```
pnbinom(5-3,3,0.5)
```

The Negative Binomial Distribution

A fair coin is tossed until the first heads comes up.

Find the probability that this takes more than 9 tosses.

The Negative Binomial Distribution

A fair coin is tossed until the first heads comes up.

Find the probability that this takes more than 9 tosses.

Solution: 0.9101562

The Negative Binomial Distribution

A fair coin is tossed until the first heads comes up.

Find the probability that this takes more than 9 tosses.

Solution: 0.9101562

```
1-pnbinom(9-3,0.5)
```

The Negative Binomial Distribution

A fair coin is tossed until the third heads comes up.

Find the probability that this takes 9 or more tosses.

The Negative Binomial Distribution

A fair coin is tossed until the third heads comes up.

Find the probability that this takes 9 or more tosses.

Solution: 0.8554688

The Negative Binomial Distribution

A fair coin is tossed until the third heads comes up.

Find the probability that this takes 9 or more tosses.

Solution: 0.8554688

```
1-pnbinom(8-3,3,0.5)
```

The Negative Binomial Distribution

A baseball player has a $.300$ batting average.

Find the probability that their third hit in a game occurs on the 5^{th} time at bat.

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Solution: 0.1875

The Negative Binomial Distribution

A baseball player has a $.300$ batting average.

Find the probability that their third hit in a game occurs on the 5^{th} time at bat.

Solution: 0.1875

```
dnbinom(5-3,3,0.5)
```