Independence

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Otherwise, E and F are said to be **dependent**

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- A chip is drawn randomly from the urn. The chip is then replaced.
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- A chip is drawn randomly from the urn. The chip is then replaced.
- A second chip is drawn from the urn.

Now define the following events:

- The first chip drawn is blue
- The second chip drawn is blue

Independent Events

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As a result, the two events

The first draw yields a blue chip

and

The second draw yields a blue chip

are independent.

Independent Events

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In this case we say they are **dependent**

The Multiplication Rule

If two events E and F are independent, the probability that **both** occur is always:

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That is, the probability that **all** of the events $E, F, G, \ldots$ occur is always the product of the individual probabilities.

The Multiplication Rule

Example: A coin is tossed, and then a die is rolled.

If E is the event that the coin comes up heads, and F is the event that a 6 is rolled,

what is the probability of the event $(E$ and $F)$, (heads and a 6)?

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what is the probability of the event $(E \text{ and } F)$, (heads and a 6)?

- The probability of event E is $\frac{1}{2}$
- The probability of event F is $\frac{1}{6}$
- E and F are independent, so $P(E \text{ and } F) = P(E) \cdot P(F) = \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$