Compliments and Disjoint Events

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In the card draw, the compliment of "A face card" is the die set of 40 non-face cards.

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Sometimes the compliment E^c is easier to compute than E, and some times E is easier.

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To compute the probability of the event "A number less than 6 is rolled", we could add the probabilities of the five numbers 1,2,3,4,5:

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The identity

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allows us to always compute the easier probability.

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These are aggregates of elements in the sample space, but are not considered to be part of the sample space themselves.

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Two disjoint events cannot occur in the same trial of an experiment, so for disjoint events,

$$P(E \text{ or } F) = P(E) + P(F)$$

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For disjoint events,

$$P(E \text{ and } F) = 0$$

so the two formulas are consistent.