Conditional Probability

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If we know that the first card drawn is a king, there are then 3 kings in the 51 cards left, so the probability that the second card is a king is:

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This is called the **conditional probability that the second card is a king, given that the first is a king**.

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In this case events like "an even digit on the first ball" and "an even digit on the second ball" are not independent.

Multiplication Rule

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Because the events are independent, the probability of a 6 on the first die **and** a 5 on the second is:

$$\frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$$

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Knowing that the number is odd changes the sample space, eliminating 2,4, and 6.

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If we have equally likely outcomes, we can state this in terms of the number of outcomes belonging to E and F:

$$P(F|E) = \frac{N(E \text{ and } F)}{N(E)}$$

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That is, the probability that both E and F occur is always the probability that E occurs, times the conditional probability that F occurs given that E occurred.