Computing Probabilities

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- The Classical Method
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We will examine the characteristics and strengths of each of them.

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Example: We are interested in the probability that Adrian Beltre gets a hit when he bats.

According to the Boston Red Sox web site, to date in 2010 Adrian has:

- batted 576 times
- gotten 173 hits

The Empirical Method

By definition, the approximate probability $P(E)$ of the event "Adrian Beltre gets a hit when he bats" is computed as:

$$P(E) = \frac{\text{number of times he got a hit}}{\text{number of times he batted}} = \frac{173}{576} = .326$$

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In general, the empirical method calculates the approximate probability as:

$$P(E) = \frac{\text{number of times the event occurred}}{\text{number of times the experiment was performed}}$$

The Empirical Method

The Empirical Method has the following characteristics:

- It requires *data* consisting of a historical record of the results of the experiment.
- It does not require any assumptions about the underlying probability distribution.
- It gives an *approximate* value of the probability of the event.
- Its use is justified by the law of large numbers.

The Empirical Method

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Using the definition, the probability $P(E)$ that a flight is on time is:

$$P(E) = \frac{\text{number of flights arriving on time}}{\text{number of flights operated}} = \frac{893}{972} = .919$$

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Example: We are interested in the probability that a single card drawn from a shuffled deck of 52 cards is a club.

We know there are 13 clubs in the deck. According to the Classical Method, the probability of a club is:

$$P(E) = \frac{\text{number of clubs}}{\text{number of cards}} = \frac{13}{52} = .25$$

The Classical Method

The Classical Method has the following characteristics:

- It requires that the experiment have all possible outcomes equally likely.
- It does not require the experiment to actually be performed.
- It gives an *exact* value of the probability of the event.

The Classical Method

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Use the Classical Method to compute the probability that a number between 1 and 100 (inclusive) is drawn.

The event occurs if one of the 100 numbers between 1 and 100 is drawn. By definition, the Classical Method gives:

$$P(E) = \frac{\text{number of results between 1 and 100}}{\text{number of possible results}} = \frac{100}{1000} = .100$$

The Subjective Method

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Although the result of the subjective method is a guess, presumably the person making the estimate is knowledgeable about the experiment.

In many cases, an estimate of the probability of the event is required, but no data is available and the assumption of equally likely outcomes is not reasonable.

This means we cannot use the empirical method or the classical method, so the subjective method is our only choice.

The Subjective Method

Example: In the WASH study of the safety of a nuclear power plant design, the author estimated that the probabiliity that a certain valve was closed by accident when it should have been left open as one in 1,000 (0.001).

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So a if a horse is given $8:1$ odds of winning, the bookmaker has estimated that if the horse is entered in nine races, it will lose 8 times and win once. This corresponds to a subjective probability of the horse winning as:

$$P(E) = \frac{1}{1+8} = \frac{1}{9} = .111$$

Examples

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If 23,450 cars of this make and model were sold, how would you estimate the probability that the part fails in the first 50,000 miles of use?

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If 23,450 cars of this make and model were sold, how would you estimate the probability that the part fails in the first 50,000 miles of use?

Solution: The empirical method is applicable, with

$$P(E) = \frac{\text{number of times the part failed}}{\text{number of cars sold}} = \frac{1823}{23450} = .078$$

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How would you estimate the probability that 11-11-11 actually comes up on this date?

Solution: The classical method applies, because we have $100 \times 100 \times 100 = 1,000,000$ equally likely outcomes:

$$P(E) = \frac{\text{number of times 11-11-11 comes up}}{\text{number of possible outcomes}} = \frac{1}{1,000,000}$$

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Solution: This is a subjective estimate, and the probability is

$$P(E) = \frac{\text{number of times Red Sox win}}{\text{total trials}} = \frac{1}{18+1}$$

$$= \frac{1}{19} = .0526$$