Addition Rules

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We have introduced methods of computing probabilities that allow us to compute probabilities such as, for the card draw experiment,

$$P(\text{A spade is drawn}) = \frac{\text{number of spades}}{\text{number of cards}} = \frac{13}{52}$$

or

$$P(\text{An ace is drawn}) = \frac{\text{number of aces}}{\text{number of cards}} = \frac{4}{52}$$

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We do not yet have a way to determine the probabiliity of a combined event like

$$P(\text{A spade is drawn or an ace is drawn})$$

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There are 13 spades that qualify, and once we have accounted for them, we have to include the three aces that **are not spades**. This gives us a total of 16 cards.

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At first glance it would seem that if we want to find the number of ways we can get "a spade or an ace", we should take the number of spades and add the number of aces.

This is partly true, but not the whole story. We have to be careful that, when we combine the cards that qualify for the events "a spade" and "an ace", we do not count anything twice.

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The reason we didn't get 16 is that we counted the ace of spades twice: once as a spade, and once as an ace.

Anytime the two events have some cards in common, this will happen and adding the cards in each event will overstate the count in the combined event, called the **union** of the two events.

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This means we can correct for the double counting of the ace of spades by subtracting the number of cards we double counted, one, from the original total of 17.

It turns out that this always works. The number of outcomes that are double counted is exactly the number of outcomes that belong to both events.

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We will denote the event "a spade is drawn" by A and "an ace is drawn" by B. The number of cards that qualify for these events will be denoted

$$n(A) = 13 \quad \text{and} \quad n(B) = 4$$

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The number of cards that qualify for either event, or both events will be denoted by $n(A \text{ or } B)$. In this case, $n(A \text{ or } B) = 16$ as we have seen.

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We can formalize the idea that we add the number of cards in each event, then subtract the ones we double counted as:

$$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$$

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In this case of the "spades or ace" event,

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$$n(A \text{ or } B) = 16, \quad n(A) = 13, \quad n(B) = 4, \quad n(A \text{ and } B) = 1$$

Plugging these numbers into the formula we get

$$n(A \text{ or } B) = 13 + 4 - 1 = 16$$

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Many people think that all of mathematics was worked out long ago, but in fact the body of knowledge know as mathematics is continually growing and evolving.