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An obvious solution is to just interchange the roles of x and y. The graph of $y = x^2$ is identical to the graph of $x = \sqrt{y}$

So we can just relabel the axes and consider rotating $y = \sqrt{x}$ about the x-axis between $x = 0$ and $x = 1$.

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This works as long as we can convert the function $y = f(x)$ into $x = g(y)$, but this may not be easy.

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The difference in the two volumes is:

$$V_2 - V_1 = \pi h(r_2^2 - r_1^2) \; = \; \pi h(r_2 + r_1)(r_2 - r_1)$$

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We can simplify the expression

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This is the volume of a cylindrical shell with height h, average radius r, and thickness Δr.

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The instantaneous rate of change of the volume with respect to x is the surface area of that cylinder, so

$$\frac{dV}{dx} = 2\pi x \cdot f(x) \quad \text{and so} \quad V = 2\pi \int_a^b x \cdot f(x)\, dx$$

Question 1

The integral representing the area under the curve $y = x^2$ from $x = 0$ to $x = 1$ revolved around the y-axis is:

1. $2\pi \int_0^1 x^3 dx$
2. $2\pi \int_0^1 \pi x^2 dx$
3. $2\pi \int_0^1 x dx$
4. $\pi \int_0^1 x^3 dx$
5. $\pi \int_0^1 x^2 dx$
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Question 2

The integral representing the area under the curve $y = 1/x$ from $x = 1$ to $x = 2$ revolved around the y-axis is:

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2. $2\pi \int_1^2 1/x^2 dx$
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4. $2\pi \int_0^1 dx \ = \ 2\pi$

Question 3

The integral representing the area between the curves $y = x^3$ and $y = x$ from $x = 0$ to $x = 1$ revolved around the y-axis is:

1. $2\pi \int_0^1 x^4 dx$
2. $2\pi \int_0^1 (x^2 - x^4) dx$
3. $2\pi \int_0^1 (x^3 - x) dx$
4. $2\pi \int_0^1 (x^4 - x^2) dx$
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2. $2\pi \int_0^1 (x^2 - x^4) dx \ = \ 4\pi/15$