MTH126 Final Exam Crib Sheet

1. Integration Formulas

Constants of integration have been omitted.

$$\int \sec^2 x \, dx = \tan x \tag{1}$$

$$\int \csc^2 x \, dx = -\cot x \tag{2}$$

$$\int \sec x \tan x \, dx = \sec x \tag{3}$$

$$\int \csc x \cot x \, dx = -\csc x \tag{4}$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| \tag{5}$$

$$\int \csc x \, dx = \ln|\csc x - \cot x| \tag{6}$$

$$\int \tan x \, dx = \ln|\sec x| \tag{7}$$

$$\int \cot x \, dx = \ln|\sin x| \tag{8}$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) \tag{9}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) \tag{10}$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x - a}{x + a}\right| \tag{11}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| \tag{12}$$

2. Formulas for Exam 1

$$\int_1^\infty 1/x^p \qquad \text{converges if } p > 1 \text{ diverges otherwise} \tag{13}$$

$$\sqrt{a^2 - x^2} \quad use \quad a\sin\theta \quad -\pi/2 \le \theta \le \pi/2 \tag{14}$$

$$\sqrt{a^2 + x^2} \quad use \quad a\tan\theta \quad -\pi/2 \le \theta \le \pi/2 \tag{15}$$

$$\sqrt{x^2 - a^2} \quad use \quad a\sec\theta \quad 0 \le \theta \le \pi/2 \tag{16}$$

$$\sin A \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)] \tag{17}$$

$$\sin A \sin B = \frac{1}{2}[\cos(A-B) - \cos(A+B)] \tag{18}$$

$$\cos A \cos B = \frac{1}{2}[\cos(A-B) + \cos(A+B)] \tag{19}$$

$$\cos^2\theta = \frac{1}{2}[1 + \cos 2\theta] \tag{20}$$

$$\sin^2\theta = \frac{1}{2}[1 - \cos 2\theta] \tag{21}$$

$$|E_S| \le K(b-a)^5/180n^4 \quad where \quad |f^{(4)}(x)| \le K \text{ for } a \le x \le b \tag{22}$$

$$|E_T| \le K(b-a)^3/12n^2 \quad where \quad |f''(x)| \le K \text{ for } a \le x \le b \tag{23}$$

$$|E_M| \le K(b-a)^3/24n^2 \quad where \quad |f''(x)| \le K \text{ for } a \le x \le b \tag{24}$$

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3. Formulas for Exam 2

Taylor series expansion centered at $x = a$:

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \qquad |x-a| < R$$

$$= f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f^{(4)}(a)}{4!}(x-a)^4 + \cdots$$

Maclaurin series expansion (special case of Taylor series with $a = 0$):

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n \qquad |x-a| < R$$

$$= f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{(4)}(0)}{4!}x^4 + \cdots$$

Binomial series: If k is any real number and $|x| < 1$,

$$(1+x)^k = \sum_{n=0}^{\infty} \binom{k}{n} x^n$$

$$= 1 + kx + \frac{k(k-1)}{2!}x^2 + \frac{k(k-1)(k-2)}{3!}x^3 + \frac{k(k-1)(k-2)(k-3)}{4!}x^4 + \cdots$$

Maclaurin series expansion for e^x

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!}x^n \qquad x \in (-\infty, \infty)$$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

4. Formulas for Exam 3

The general solution for the first-order linear differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

is

$$y(x) = \frac{1}{I(x)} \left[\int I(x)Q(x)dx + C \right]$$

where

$$I(x) = \exp\left(\int P(x)dx \right)$$

The solution to the logistic equation

$$\frac{dP}{dx} = kP\left(1 - \frac{P}{K}\right)$$

is

$$P(t) = \frac{K}{1 + Ae^{-kt}} \quad \text{wheere} \quad A = \frac{K - P(0)}{P(0)}$$

Recursion formulas for Euler's method:

$$x_{n+1} = x_n + h \quad y_{n+1} = y_n + y'_n \cdot h$$

Determine y_n' by substituting y_n and x_n into the differential equation.

General solution of homogeneous second order linear differential equation with constant coefficients:

If the auxiliary equation has two real roots r_1 and r_2 the general solution is

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

If the auxiliary equation has one real root r the general solution is

$$y = c_1 e^{rx} + c_2 x e^{rx}$$

If the auxiliary equation has complex roots $\alpha+\beta i$ and $\alpha-\beta i$ the general solution is

$$y = e^{\alpha x}\left(c_1 \cos\beta x + c_2 \sin\beta x\right)$$

5. Additional Formulas for the Final

The length L of a curve $y = f(x)$ from $x = a$ to $x = b$ is given by

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

The surface area of the surface obtained by rotating the curve $y = f(x)$ from $x = a$ to $x = b$ about the x-axis is given by

$$S = \int_a^b 2\pi f(x)\sqrt{1 + [f'(x)]^2}\,dx$$