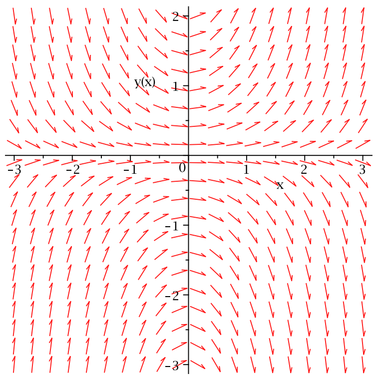
MA126 Exam 3 Version 1

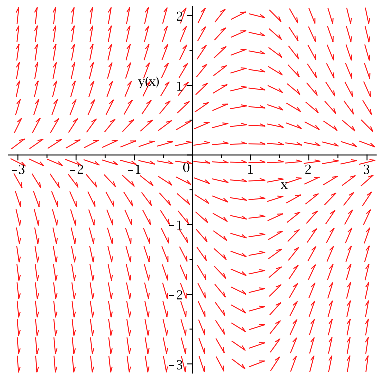
Name:

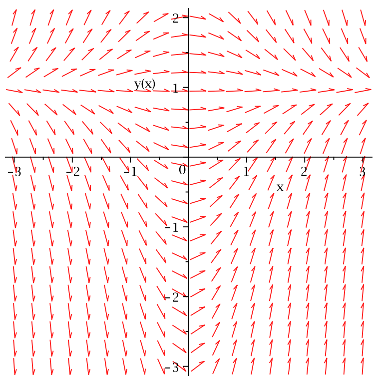
1) Match the direction field to the differential equation that best fits:

Eqn:( )

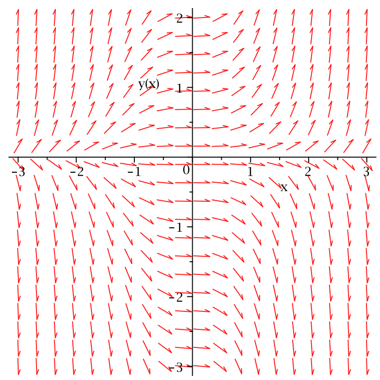


Eqn:( )





Eqn:( )



Eqn:( )

a) $y' = x^2y$

b) $y' = x(2 - y)$

c) $y' = y(1 - x)$

d) $y' = xy$

e) $y' = y(2 + x)$

f) $y' = x(1 - y)$

2) A tank contains $1000L$ of pure water with $5kg$ of dissolved salt. Brine containing $0.05kg$ of salt per liter is pumped in at a rate of $10L/min$. The solution is kept thoroughly mixed and $10L/min$ is removed from a drain at the bottom of the tank. Find an equation for the amount of salt in the tank after t minutes.

3) Find an equation of the curve that passes through the point $(1, 1)$ whose slope at the point (x, y) is

$$\frac{\sqrt{y}}{x}$$

4) Solve the initial value problem:

$$xy' + y = \frac{1}{\sqrt{x}}, \qquad y(1) = 1$$

5) Solve the initial value problem:

$$y' + \frac{y}{2} = e^t, \quad y(1) = 0$$

6) Birds and insects are modeled by the equations

$$\frac{dx}{dt} = 0.4x - 0.002xy \qquad (1)$$

$$\frac{dy}{dt} = -0.2y + 0.000008xy \qquad (2)$$

Find the equilibrium solution(s), if there are any.

7) Solve the differential equation

$$y'' - 12y' + 4y = 0$$

8) Solve the boundary-value problem

$$y'' + 2y' + 2y = 0, \quad y(0) = 2, \quad y'(0) = 1$$

9) Given the initial-value problem

$$y' = x - y, \quad y(0) = 1$$

Use Euler's method with a step size of 0.4 to compute the approximate value of $y(1.2)$

10) A unknown (but positive) number of flour beetles take up residence in a container of flour. The beetle population grows according to the following equation:

$$P(t) = \frac{1100}{1 + 10e^{-0.02t}}$$

a) What value does the beetle population approach as t becomes large?

b) What is the initial size of the beetle population?

The general solution for the first-order linear differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)$$

is

$$y(x) = \frac{1}{I(x)}\left[\int I(x)Q(x)dx + C\right]$$

where

$$I(x) = \exp\left(\int P(x)dx\right)$$

The solution to the logistic equation

$$\frac{dP}{dx} = kP\left(1 - \frac{P}{K}\right)$$

is

$$P(t) = \frac{K}{1 + Ae^{-kt}} \quad \text{wheere} \quad A = \frac{K - P(0)}{P(0)}$$

Recursion formulas for Euler's method:

$$x_{n+1} = x_n + h \quad y_{n+1} = y_n + y_n' \cdot h$$

Determine y_n' by substituting y_n and x_n into the differential equation.

General solution of homogeneous second order linear differential equation with constant coefficients:

If the auxiliary equation has two real roots r_1 and r_2 the general solution is

$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x}$$

If the auxiliary equation has one real root r the general solution is

$$y = c_1 e^{rx} + c_2 x e^{rx}$$

If the auxiliary equation has complex roots $\alpha + \beta i$ and $\alpha - \beta i$ the general solution is

$$y = e^{\alpha x} \left(c_1 \cos \beta x + c_2 \sin \beta x \right)$$

Integration Formulas

Constants of integration have been omitted.

$$\int \sec^2 x \, dx = \tan x \tag{3}$$

$$\int \csc^2 x \, dx = -\cot x \tag{4}$$

$$\int \sec x \tan x \, dx = \sec x \tag{5}$$

$$\int \csc x \cot x \, dx = -\csc x \tag{6}$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| \tag{7}$$

$$\int \csc x \, dx = \ln |\csc x - \cot x| \tag{8}$$

$$\int \tan x \, dx = \ln |\sec x| \tag{9}$$

$$\int \cot x \, dx = \ln |\sin x| \tag{10}$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) \tag{11}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left(\frac{x}{a} \right) \tag{12}$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| \tag{13}$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left| x + \sqrt{x^2 \pm a^2} \right| \tag{14}$$