MTH125 Stewart Section 1.5

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Exponential Functions

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The definition requires that a be **strictly positive**

Exponential Functions

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The range of the exponential function $f(x) = a^x$ is all *positive* real numbers,

$$\{x \mid x > 0\}$$

or

$$(0, \infty)$$

in interval notation.

Exponential Functions

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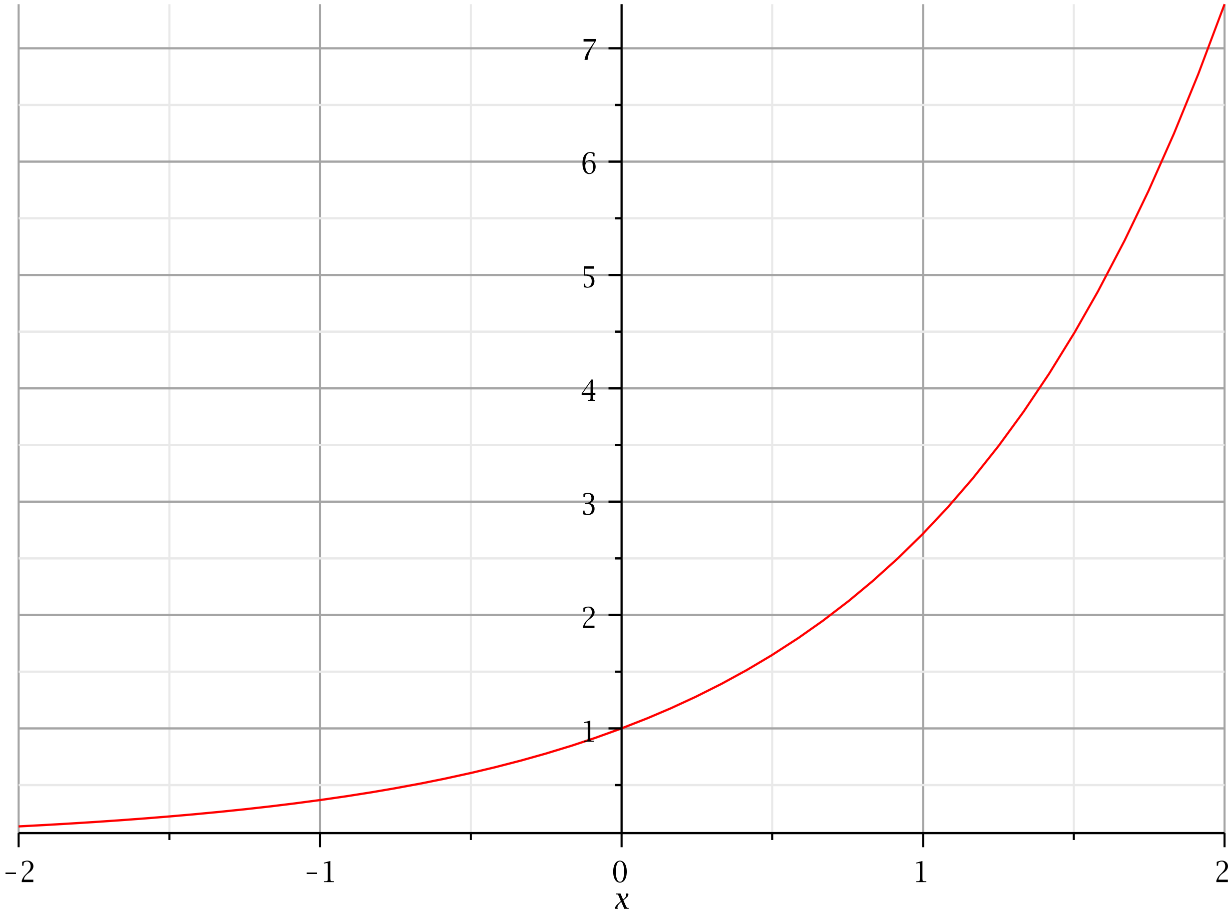
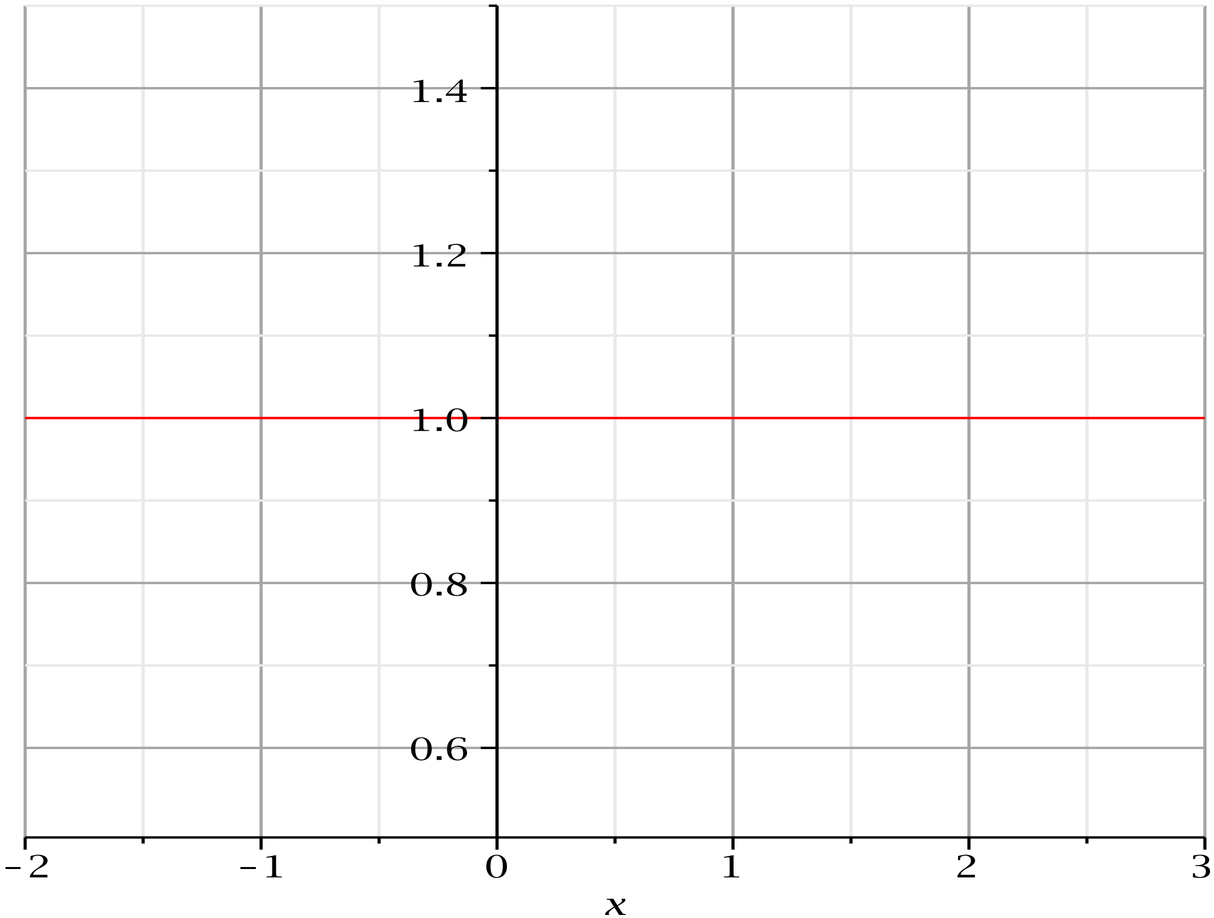
When $a > 1$, the graph approaches the x-axis as x tends to $-\infty$, and tends to $+\infty$ for large positive values of x.



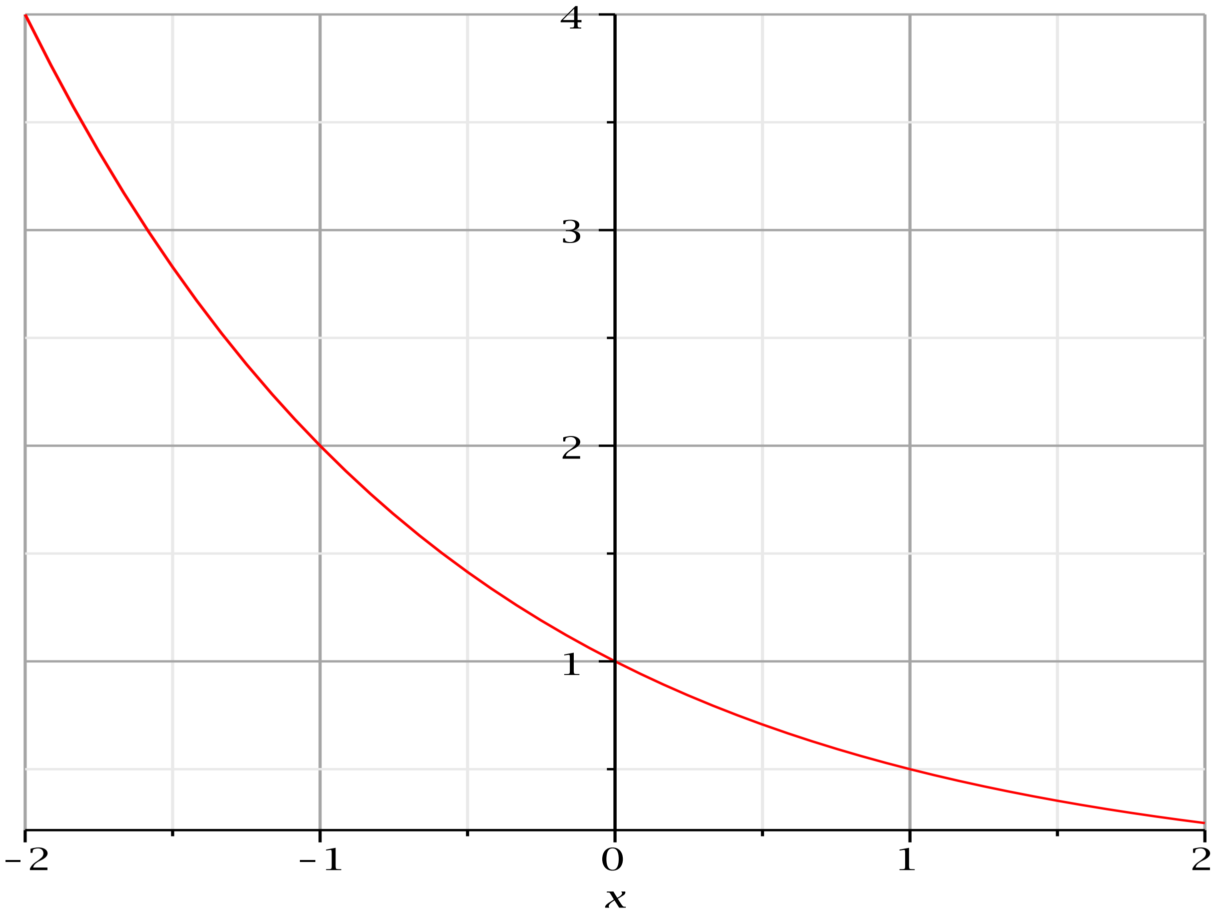
Exponential Functions

When $a = 1$, not a very interesting case, the graph is a horizontal line:



Exponential Functions

When $a < 1$, the graph approaches the x-axis as x tends to ∞, and tends to $+\infty$ for large **negative** values of x.



Exponential Functions

Since

$$a^0 = 1$$

the value of any exponential function at $x = 0$ is 1.

So, the graph of every exponential function passes through the point (1,0)

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The most useful number to use as a base for an exponential function is a transcendental number called e, which has a value of about 2.718.

Exponential Functions

The function

$$f(x) = e^x$$

has the property that the slope of the line tangent to the curve at the point

$$(x, e^x)$$

is e^x, the same value as $f(x)$.

Exponential Functions - Sample Proble

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Every increasing exponential function has the property that the time required for $f(x)$ to double is constant, regardless of the value of x.

Exponential Functions - Sample Proble

One application of exponential functions is the growth of tumors. Oncologists refer to tumors as being more aggressive or less aggressive, and sometimes use the doubling time (i.e., the time for a tumor to double in size) as a measure of aggressiveness.

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For example, if a tumor is small and has a doubling time of 10 years, the patient may succumb to old age before the tumor becomes life-threatening. On the other hand, an aggressive tumor might have a doubling time of two weeks or less.

Exponential Functions - Sample Proble

Suppose the number of cells in a tumor after time t is given by

$$p(t) = p_0 e^{Kt}$$

and is known to double every 90 days. What is the value of K?

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Evaluate $p(t)$ at t and $t + 90$:

$$p(t) = p_0 e^{Kt} \quad p(t+90) = p_0 e^{Kt}$$

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or

$$2 = \frac{p_0e^{K(t+90)}}{p_0e^{Kt}} = \frac{e^{K(t+90)}}{e^{Kt}} = e^{K\cdot 90}$$

Exponential Functions - Sample Proble

Since the logarithm is a $1:1$ function, we can take the log of both sides to get

$$\ln(2) = \ln(e^{K\cdot 90}) = K \cdot 90$$

so

$$K = \frac{\ln 2}{90}$$

Exponential Functions - Sample Proble

An equivalent but easier way is the following: Since the cell count doubles every 90 days, we can pick any 90 day interval we want. Choose to $t = 0$ and $t = 90$ to get the simpler equation

$$2p_0 = p_0 e^{K \cdot 90}$$

and as before divide by p_0 and take logs to get

$$\ln 2 = \ln(e^{K \cdot 90}) = K \cdot 90 \quad \Rightarrow \; K = \frac{\ln 2}{90}$$