

Question 1

If a function is continuous at $x = a$,
it is also differentiable at $x = a$

1. TRUE
2. FALSE

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If a function is continuous at $x = a$,
it is also differentiable at $x = a$

1. TRUE
2. FALSE

The statement is false.

Differentiable implies continuous, but not the other way around.

$|x|$ is continuous, but not differentiable at $x = 0$.

Question 2

The limit

$$\lim_{x \rightarrow a} f(x) = L$$

can exist even if $f(a)$ is not defined.

1. TRUE
2. FALSE

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The limit

$$\lim_{x \rightarrow a} f(x) = L$$

can exist even if $f(a)$ is not defined.

1. TRUE
2. FALSE

The statement is true.

The definition of a limit is very carefully worded so that it says nothing at all about $f(a)$.

Question 3

The function

$$f(x) = \frac{1}{\sqrt{x^2 - 1}}$$

is continuous on $(-1, 1)$

1. TRUE
2. FALSE

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The function

$$f(x) = \frac{1}{\sqrt{x^2 - 1}}$$

is continuous on $(-1, 1)$

1. TRUE 2. FALSE

The statement is false.

Roots are continuous on their domains, and quotients are continuous on the intersection of the domains of the numerator and denominator, excluding points where the denominator is zero.

This function is continuous on $(-\infty, -1) \cup (1, \infty)$.

Question 4

The function

$$f(x) = \sqrt{4 - x^2}$$

is continuous on $[-2, 2]$

1. TRUE
2. FALSE

Question 4

The function

$$f(x) = \sqrt{4 - x^2}$$

is continuous on $[-2, 2]$

1. TRUE 2. FALSE

The statement is true.

Roots are continuous on their domains. For an even root, the domain is the set of x values for which the quantity under the radical is nonnegative, that is,

$$\{x : 4 - x^2 \geq 0\} = [-2, 2]$$

Question 5

Given that f is continuous on $(-1, 1)$ and

$$f(-1) = 1 \quad \text{and} \quad f(1) = -1$$

the intermediate value theorem guarantees that f assumes the value 0 somewhere in the interval $(-1, 1)$.

1. TRUE
2. FALSE

Question 5

Given that f is continuous on $(-1, 1)$ and

$$f(-1) = 1 \quad \text{and} \quad f(1) = -1$$

the intermediate value theorem guarantees that f assumes the value 0 somewhere in the interval $(-1, 1)$.

1. TRUE
2. FALSE

The statement is false.

The IVT requires:

- f is continuous on the **closed** interval $[a, b]$
- $f(a) \neq f(b)$

Question 6

We say

$$\lim_{x \rightarrow a} f(x) = L$$

if and only if

$$\lim_{x \rightarrow a^-} f(x) = L = \lim_{x \rightarrow a^+} f(x)$$

1. TRUE
2. FALSE

Question 6

We say

$$\lim_{x \rightarrow a} f(x) = L$$

if and only if

$$\lim_{x \rightarrow a^-} f(x) = L = \lim_{x \rightarrow a^+} f(x)$$

1. TRUE 2. FALSE

The statement is true.

The two-sided limit exists only if the left and right hand limits both exist and are the same real number L .

Question 7

A function f can be left continuous at $x = a$, even if $f(a)$ is undefined.

1. TRUE
2. FALSE

Question 7

A function f can be left continuous at $x = a$, even if $f(a)$ is undefined.

1. TRUE 2. FALSE

The statement is false.

A function is left continuous at $x = a$ if

$$\lim_{x \rightarrow a^-} f(x) = f(a)$$

so $f(a)$ has to exist.

Question 8

If a function is continuous at $x = a$,
then it has the direct substitution property at $x = a$.

1. TRUE
2. FALSE

Question 8

If a function is continuous at $x = a$,
then it has the direct substitution property at $x = a$.

1. TRUE 2. FALSE

The statement is true.

Continuity implies the direct substitution property.

Question 9

If a function is continuous at $x = a$,
then it is also left and right continuous at $x = a$.

1. TRUE
2. FALSE

Question 9

If a function is continuous at $x = a$,
then it is also left and right continuous at $x = a$.

1. TRUE 2. FALSE

The statement is true.

Continuity *at a point* implies both left and right continuity at that point.

Question 10

We say a function is continuous on an interval $[a, b]$ if it is:

- right continuous at $x = a$
- left continuous at $x = b$
- continuous for all $x \in (a, b)$.

1. TRUE
2. FALSE

Question 10

We say a function is continuous on an interval $[a, b]$ if it is:

- right continuous at $x = a$
- left continuous at $x = b$
- continuous for all $x \in (a, b)$.

1. TRUE 2. FALSE

The statement is true.

This is the definition of continuity on an interval.