MA125 Exam 3 Version 1

Name:

***1)** Find two numbers x and y whose sum is 1 that make

$$x^2 + 4y$$

is as small as possible.

***2)** We want to find the critical numbers of

$$f(x) = \frac{x^4}{4} - \frac{x^2}{2} - 2x$$

on the interval $[1, 3]$. If Newton's method is used to find the points where $f'(x) = 0$ with a starting value $x_0 = 1$, what is x_2?

***3)** Find the absolute maximum and minimum of the function

$$f(x) = x\sqrt{1-x} \quad \text{on} \quad [-1, 1]$$

***4)** Find the point on the graph of

$$f(x) = \sqrt{x+1}$$

that is closest to the origin.

***5)** Find

$$\lim_{x \to 0} \frac{e^{4x} - 1 - 4x}{x^2}$$

***6)** Find the area between the graph of

$$f(x) = 3x^2 + 2x + 5$$

and the x-axis between 0 and 3.

***7)** Find the most general antiderivative of

$$f(x) = xe^x + e^x$$

***8)** Find $g'(x)$ if a is a constant and

$$g(x) = \int_a^x \frac{t-a}{\sqrt{t^2-a^2}}\,dt$$

Suppose

$$f(x) = 2x^3 + 3x^2 - 12x + 3$$

9a) Which of the following lists contains **all** of the intervals on which f is increasing?

a) $(-\infty, -2)$,$(1/2, \infty)$
b) $(-2, 1)$
c) $(-\infty, -2), (1, \infty)$
d) $(-1, 2)$
e) $(-\infty, \infty)$

9b) Which (if any) of the following conclusions can we draw from Rolle's Theorem?

a) $f'(c) = 0$ for some $c \in (1, 2)$
b) $f'(c) = 0$ for some $c \in (0, 2)$
c) $f'(c) = 0$ for some $c \in (0, 1)$
d) $f'(c) = 0$ for some $c \in (-2, 1)$
e) None of the above.

9c) Which of the following lists contains **all** of the intervals on which f is concave up?

a) $(-1, \infty)$
b) $(-1/2, \infty)$
c) $(-\infty, -1/2), (1, \infty)$
d) $(-1/4, 1)$
e) $(-\infty, -1/2)$

9d) Which (if any) of the following conclusions can we draw from the Mean Value Theorem?

a) $f'(c) = 16$ for some $c \in (-2, 1)$
b) $f'(c) = 3$ for some $c \in (-2, 1)$
c) $f'(c) = 7$ for some $c \in (-2, 1)$
d) $f'(c) = -9$ for some $c \in (-2, 1)$
e) None of the above, the theorem does not apply

9e) If g is a function with the property that $f'(x) = g'(x)$ for $x \in (-2, 1)$, which of the following conclusions can be drawn?

a) The line tangent to f at $x = 1$ intersects the line tangent to g at $x = 2$
b) Secant lines to the graph of f and g drawn with $x_1 = 1$ and $x_2 = 2$ are parallel
c) The function $h(x) = f(x) - g(x)$ has a value of zero on $(-1, 1)$
d) $f(0) = g(0)$
e) None of the above

***10)** Find a function f that satisfies:

$$f'(x) = x^2 + \frac{1}{x} \quad \text{and} \quad f(1) = 2$$